MECHANICAL DRAWING

PART III

AMERICAN SCHOOL of CORRESPONDENCE
CHICAGO ILLINOIS

MECHANICAL DRAWING

PART III.

INSTRUCTION PAPER

PREPARED BY

ERVIN KENISON, S. B.,

ASSISTANT PROFESSOR OF MECHANICAL DRAWING AND DESCRIPTIVE GEOMETRY,
MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

AMERICAN SCHOOL OF CORRESPONDENCE

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MECHANICAL DRAWING

PART III

PROJECTIONS

ORTHOGRAPHIC PROJECTION

Principles. Orthographic projection is the art of representing objects by casting their outlines upon two planes at right angles to each other, so that from the outlines alone the forms and positions of the objects may be determined. The planes of projection are pictured in Fig. 95, one plane, H, being horizontal, and the other, V, vertical. The intersections of these planes are a straight line GL, known as the *ground line.*

Orthographic projection requires two distinct processes of the imagination; first, to conceive each view of the corresponding plane of projection; second, to find the position of the different views with respect to each other and to the ground line when the planes of projection have been made into one plane, Fig. 96. A draftsman becomes so accustomed to this process that he is not conscious of any action of the imagination, but pictures the different views of an object without thinking of the planes of projection. The ability to obtain adequate mental pictures of the horizontal and vertical projections of an object—called *plan* and *elevation*, respectively—must therefore be acquired by practice.

Plan and Elevation. To illustrate a case. Suppose a one-inch cube to be placed, as shown in Fig. 95, with the top face horizontal and the front face parallel to the vertical plane. Now, suppose the sun is in such a position that its rays are perpendicular to the horizontal, or H, plane, then a shadow of the cube will be cast or projected on the H plane; this shadow called the *plan*, will be a one-inch square, 1^h-2^h-3^h-4^h, Fig. 95.

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If the sun is in a position such that its rays are perpendicular to the V plane a similar square 3-4-5-6, Fig. 95, will be projected on this plane as the elevation. In order that the projected views, or projections, Fig. 96, as they are better known, may be shown on one plane or on a sheet of drawing paper, the plane H with the projection upon it is revolved about GL as an axis until it lies in the same plane as V.

Fig. 95 represents the picture and is for the purpose of showing the student how the object is projected to the horizontal and vertical

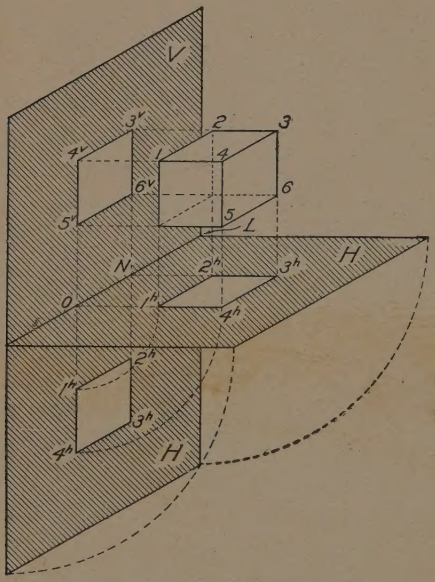

Fig. 95. Object and Its Projections

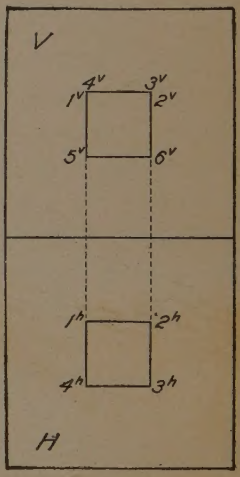

Fig. 96. Vertical and Horizontal Projections

planes. Fig. 96 represents the actual projections, plan and elevation, as the problem would be drawn. The projections of a point a and line B drawn according to the first method are shown in Fig. 97.

It may be seen from the above explanation that the projected view of a point of an object on a plane is in a perpendicular drawn to the plane through the point of the object.

In Fig. 95, the lines 1^h O and 2^h N, being perpendicular to GL, are in the same straight lines as 5^v O and 6^v N, which also are perpendicular to GL. That is—*two views of a point are always in a line*

perpendicular to GL. From this it is evident that the plan must be vertically below the elevation, point for point. Now looking directly at the two planes in the revolved position, a true orthographic projection of the cube is obtained, as shown in Fig. 96.

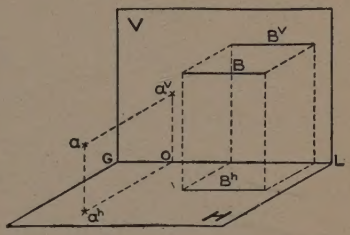

Fig. 97. Projections of a Point and a Line

All points on an object at the same height must appear in elevation at the same distance above the ground line. If numbers 1, 2, 3, and 4 on the plan, Fig. 96, indicate the top corners of the cube, then these four points, being at the same height, must be shown in elevation at the same height and at the top, $\frac{4^v}{1^v}$ and $\frac{3^v}{2^v}$. From the same figure it is seen that the top of the cube, 1-2-3-4, is shown in elevation as the straight line $\frac{4^v}{1^v}\ \frac{3^v}{2^v}$. This illustrates the fact that *if a surface is perpendicular to either plane of projection, its projection on that plane is simply a line—a straight line if the surface is plane, a curved line if the surface is curved.* From Fig. 96 it is seen that the top edge of the cube, 1-4, has for its projection on the vertical plane the point $\frac{4^v}{1^v}$, the principle of which is stated in this way: *If a straight line is perpendicular to either V or H, its projection on that plane is a point, and on the other plane is a line equal in length to the line itself, and perpendicular to the ground line.*

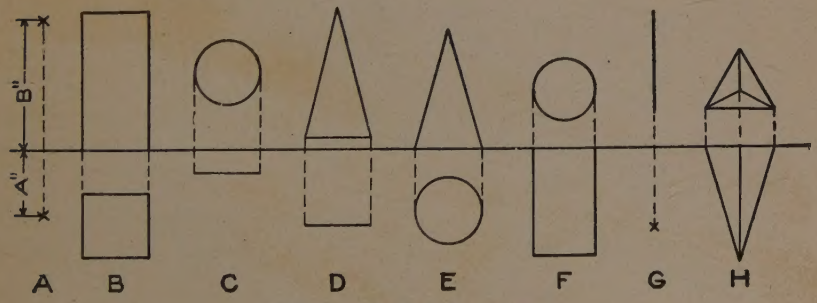

Fig. 98. Typical Projections

Typical Examples. Fig. 98 is given as an exercise to show clearly the idea of plan and elevation.

A = a point B'' above H, and A'' in front of V
B = square prism resting on H, two of its faces parallel to V
C = circular disk in space parallel to V
D = triangular card in space parallel to V
E = cone resting on its base on H
F = cylinder perpendicular to V, and with one end resting against V
G = line perpendicular to H
H = triangular pyramid above H, with its base resting against V

Suppose in Fig. 99, that it is desired to construct the projections of a prism $1\frac{1}{2}$ in. square, and 2 in. long, standing on one end on the horizontal plane, two of its faces being parallel to the vertical plane. In the first place, as the top end of the prism is a square, the top view or plan will be a square of the same size, that is, $1\frac{1}{2}$ in. Then, since the prism is placed parallel to and in front of the vertical plane, the plan, $1\frac{1}{2}$ in. square, will have two edges parallel to the ground line. As the front face of the prism is parallel to the vertical plane its projection on V will be a rectangle, equal in length and width to the length and width respectively of the prism, and as the prism stands with its base on H, the elevation, showing height above H, must have its base on the ground line. Observe carefully that points in elevation are vertically over corresponding points in plan.

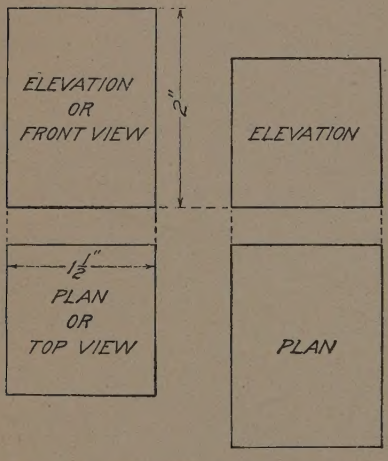

Fig. 99. Projections of a Square Prism

The second drawing in Fig. 99 represents a prism of the same size as the first but lying on one side on the horizontal plane, and with the ends parallel to V.

Summary. The principles which have been used thus far may be stated as follows:

1. If a line or point is on either plane, its other projection must be in the ground line.
2. Height above H is shown in elevation as height above the ground

line, and distance in front of the vertical plane is shown in plan as distance below the ground line.

3. If a line is parallel to either plane, its actual length is shown on that plane, and its other projection is parallel to the ground line. A line oblique to either plane has its projection on that plane shorter than the line itself, and its other projection oblique to the ground line. No projection can be longer than the line itself. If two lines intersect, their projections must cross, since the point of intersection of the lines is a point on both lines, and therefore the projections of this point must be on the projections of both lines, or at their intersection. In order that intersecting lines may be represented, the vertical projections must intersect in a point vertically above the intersection of the horizontal projections. Thus Fig. 100 represents two lines which do intersect as C^v crosses D^v at a point vertically above the intersection of C^h and D^h. In Fig. 101, however, the lines do not intersect since the intersections of their projections do not lie in the same vertical line.

4. A plane surface if parallel to either plane, is shown on that plane in its true size and shape; if oblique it is shown smaller than the true size, and if perpendicular it is shown as a straight line. Lines parallel in space must have their V projections parallel to each other and also their H projections.

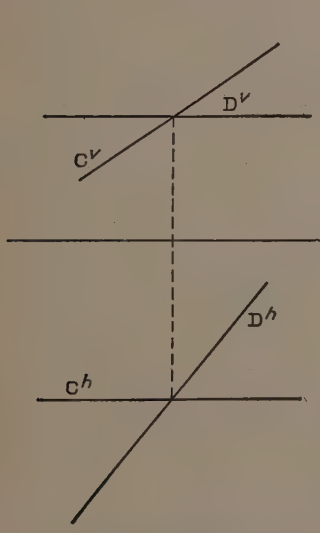

Fig. 100. Projections of Two Intersecting Lines

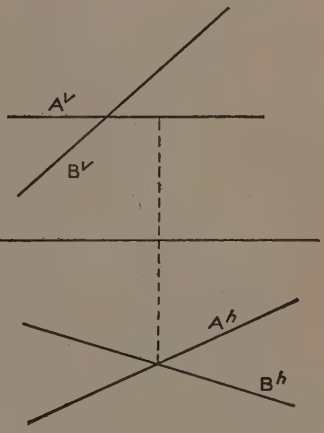

Fig. 101. Projections of Two Non-Intersecting Lines

TRUE LENGTH OF LINES

In Fig. 102 is given the plan and elevation of a square pyramid standing on the horizontal plane. The height of the pyramid is the distance $A\,B$. The slanting edges of the pyramid, $A\,C$, $A\,D$, etc., must be all of the same length, since A is directly above the

center of the base. What this length is, however, does not appear in either projection, as these edges are not parallel to either V or H.

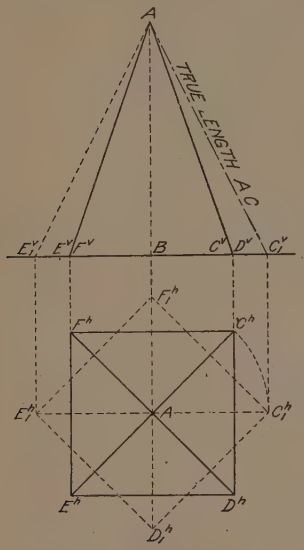

Fig. 102. Projections of Square Pyramid

Suppose the pyramid be turned around into the dotted position $C_1^h\ D_1^h\ E_1^h\ F_1^h$ where the horizontal projections of two of the slanting edges, $A\,C_1^h$ and $A\,E_1^h$, are parallel to the ground line. These two edges, having their horizontal projections parallel to the ground line, are now parallel to V, and therefore their new vertical projections will show their true lengths. The base of the pyramid is still on H, and therefore is projected on V in the ground line. The apex is in the same place as before, hence the vertical projection of the pyramid in its new position is shown by the dotted lines. The vertical projection $A\,C_1^v$ is the true length of edge AC^h. Now if it is desired to find simply the true length of $A\,C^h$, it is unnecessary to turn the whole pyramid around, as the one line $A\,C^h$ will be sufficient.

The true length of lines may therefore be found in any case by

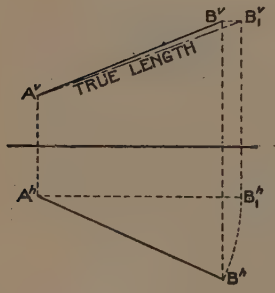

Fig. 103.

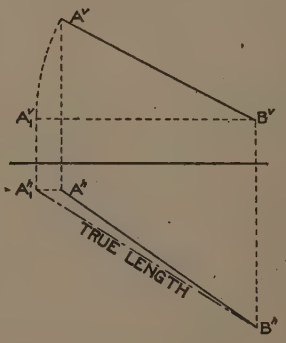

Fig. 104.

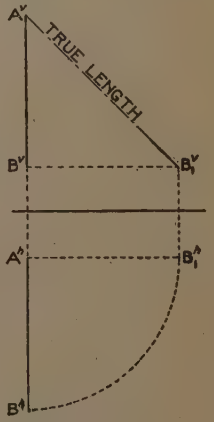

Fig. 105.

True Length of a Line by Rotating its Projection

applying the following principle: *Swing one projection of the line parallel to the ground line, using one end as center. On the other projection*

the moving end remains at the same distance from the ground line and of course vertically above or below the same end in its parallel position. This new projection of the line shows its true length, Fig. 103, Fig. 104, and Fig. 105.

PROFILE PLANE

A third plane perpendicular to both coordinate planes, and hence to the ground line, is called a *profile plane.* This plane is vertical in position, and may be used as a plane of projection. A projection on the profile plane is called a profile view, or *end view,* or sometimes edge view, and is often required in machine or other drawing when the plan and elevation do not sufficiently give the shape and dimensions.

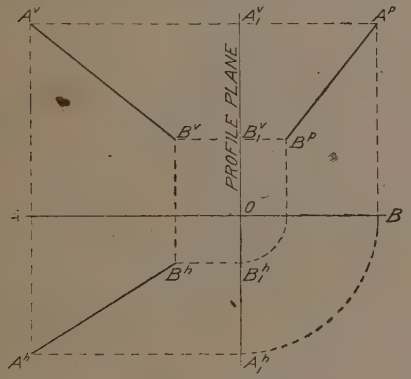

Fig. 106. Projections of a Line on Horizontal, Vertical, and Profile Planes

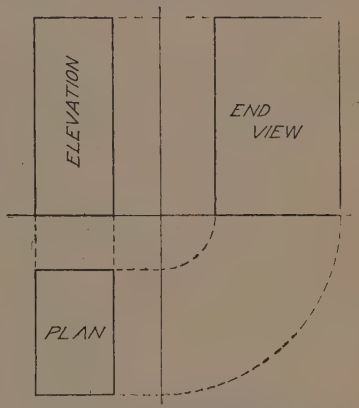

Fig. 107. Three Projections of a Rectangular Prism

A projection on this plane is found in the same way as on the V plane, *i. e.,* by perpendiculars drawn from points on the object.

Since, however, the profile plane is perpendicular to the ground line, it will be seen from the front and top simply as a straight line; in order that the size and shape of the profile view may be shown, the profile plane is revolved into V using its intersection with the vertical plane as the axis.

Given the line $A\ B$, Fig. 106, by its two projections $A^v\ B^v$ and $A^h\ B^h$, and given also the profile plane. Now by projecting the line on the profile by perpendiculars, the points $A_1^v\ B_1^v$ and $B_1^h A_1^h$ are found. Revolving the profile plane like a door on its hinges, all points in the plane will move in horizontal circles, so the horizontal projections A_1^h and B_1^h will move in arcs of circles with O as center

to the ground line, and the vertical projections B_1^v and A_1^v will move in lines parallel to the ground line to positions directly above the revolved points in the ground line, giving the profile view of the line A^p B^p. Heights, it will be seen, are the same in profile view as in elevation. By referring to the rectangular prism, Fig. 107, it is seen that the elevation gives vertical dimensions and those parallel to V, while the end view shows vertical dimensions and those perpendicular to V. The profile view of any object may be found as shown for the line A B, Fig. 106, by taking one point at a time.

REPRESENTATION OF OBJECTS

In Fig. 108 there is represented a rectangular prism or block, whose length is twice the width. The elevation shows its height. As the prism is placed at an angle, three of the vertical edges will be visible, and the fourth, invisible.

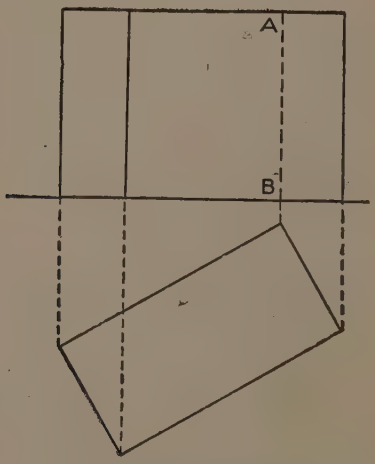

Fig. 108. Proper Representation of Rectangular Prism

In mechanical drawing the edges which in projection form a part of the outline or contour of the figure must always be visible, hence are always drawn as *full lines*, while the lines or edges which are invisible are drawn *dotted*. The plan shows what lines are visible in elevation, and the elevation determines what are visible in plan. In Fig. 108, the plan shows that the dotted edge A B is the back edge, and in Fig. 109, the elevation shows that the dotted edge C D is the lower edge of the triangular prism. In general, if in elevation an edge

projected *within* the figure is a back edge, it must be dotted, and in plan if an edge projected within the outline is a lower edge it is dotted.

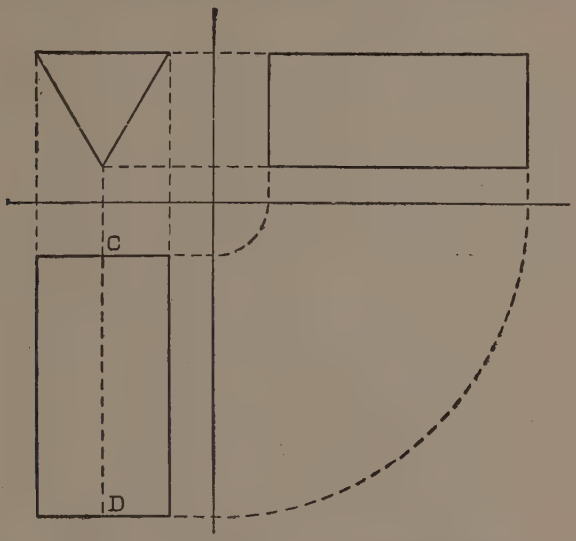

Fig. 109. Representation of Triangular Prism

Fig. 110 is a circular cylinder with its length vertical and with a hole part way through as shown in the elevation. Fig. 111 is the plan, elevation, and end view of a triangular prism with a square

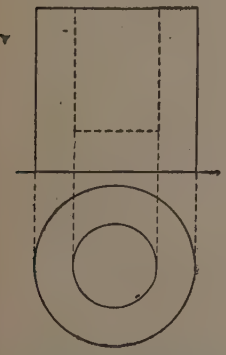

Fig. 110 Projection of Circular Cylinder

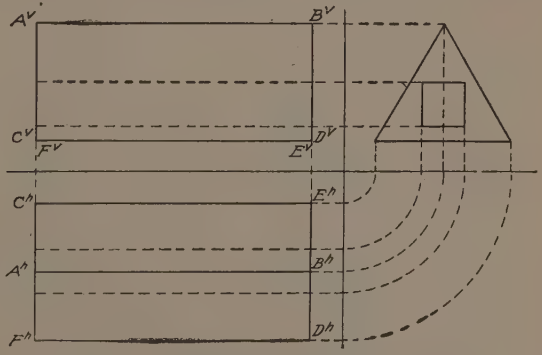

Fig. 111. Plan, Elevation, and End View of Triangular Prism

hole from end to end. The plan and elevation alone would be insufficient to determine positively the shape of the hole, but the end view shows at a glance that it is square.

ROTATION OF OBJECTS

Frustum of a Square Pyramid. In Fig. 112 is shown the plan and elevation of the frustum of a square pyramid, placed with its base on the horizontal plane. If the frustum is turned through 30°, as shown in the plan of Fig. 113, the top view or plan must still be the same shape and size, and as the frustum has not been raised or lowered, the heights of all points must appear the same in eleva-

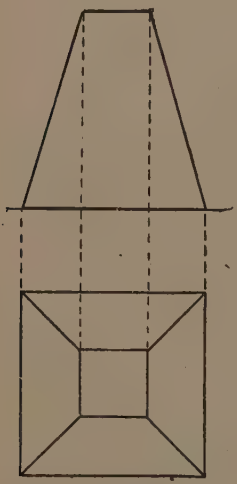

Fig. 112. Frustum of Square Pyramid

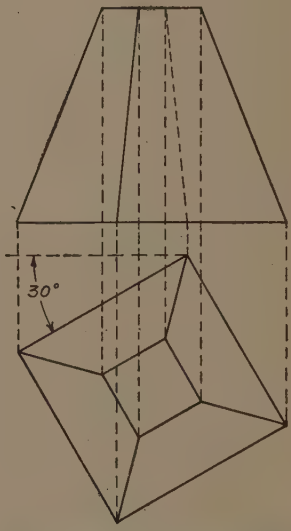

Fig. 113. Frustum of Square Pyramid Rotated Through 30°

tion as in Fig. 112. The elevation is easily found by projecting points up from the plan, and projecting the height of the top horizontally across from the first elevation.

Square Prism. The same principle is further illustrated in Figs. 114 and 115. The elevation of Fig. 114 shows a square prism resting on one edge, the lower side making an angle of 30° with the horizontal. The plan gives the true width or thickness, $\frac{5}{8}$ in., but the true length of the side, 2 in., must be obtained from the elevation, for the length of the plan is greater than 2 in. and will vary with the angle at which the prism is slanted. Now if the prism be turned around through an angle of 45° with the vertical plane, the lower edge still being on H, and the inclination of 30° with H remaining the same, the plan must remain the same size and shape, as shown

in the right-hand view of Fig. 114. The elevation is found by projecting the corners of the prism vertically to the heights of the same points in the first elevation.

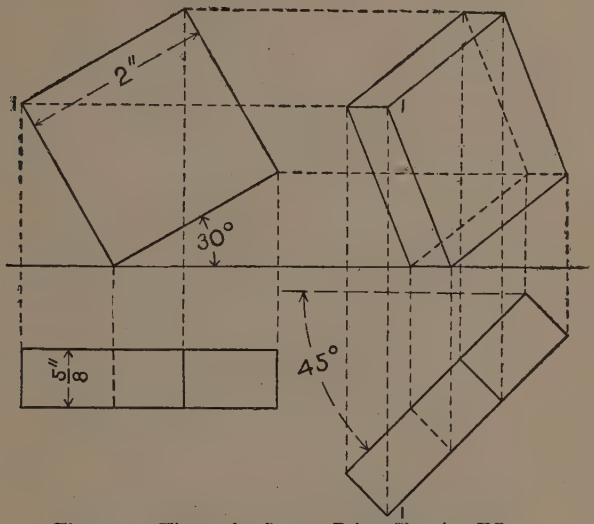

Fig. 114. Views of a Square Prism Showing Effect of Rotating Axis of Prism

Rectangular Prism. Three positions of a rectangular prism are shown in Fig. 115. In the first view, the prism stands on its

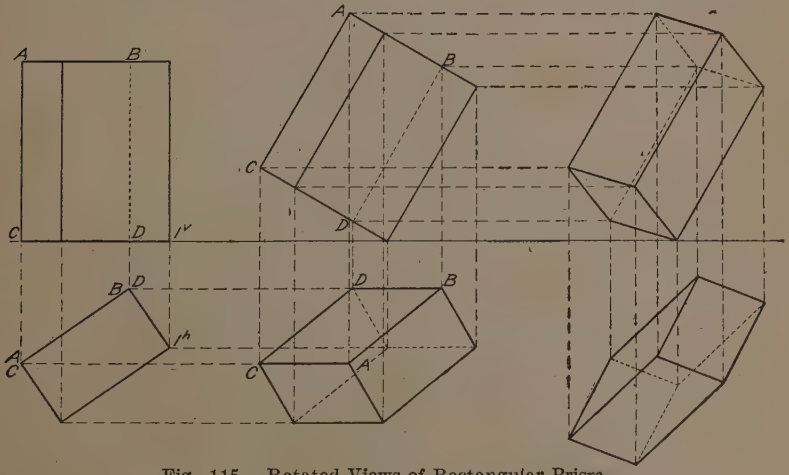

Fig. 115. Rotated Views of Rectangular Prism

base, and its axis, therefore, is parallel to the vertical plane. In the second position, the axis is still parallel to *V* and one corner of

the base is on the horizontal plane, but the prism has been turned on the line $1^h\ 1''$ as an axis, so that the inclination of all the faces of the prism to the vertical plane remains the same as before. That is, if in the first figure the side *A B D C* makes an angle of 30° with the vertical, the same side in the second position still makes 30° with the vertical plane. Hence the elevation of No. 2 is the same shape and size as in the first case. The plan is found by projecting the corners down from the elevation to meet horizontal lines projected across from the corresponding points in the first plan. The third position shows the prism with all its faces and edges making the same angles with the horizontal as in the second position, but with the plan at a different angle with the ground line. The plan then is the same shape and size as in No 2, and the elevation is found by projecting up to the same heights as shown in the preceding elevation. This principle may be applied to any solid, whether bounded by plane surfaces or curved; as far as it relates to heights, it is the same that was used for profile views.

End View a Means of Obtaining True Height. An end view is sometimes necessary before the plan or elevation of an object can be drawn. Suppose that in Fig. 116 it is desired to draw the plan and elevation of a triangular prism 3 inches long, the end of which is an equilateral triangle $1\frac{1}{2}$ inches on each side. The prism is lying on one of its three faces on *H*, and inclined toward the vertical plane at an angle of 30°. The plan may be drawn at once, because the width will be $1\frac{1}{2}$ inches, and the top edge will be projected half way between the other two. The length of the prism will also be shown. Before the elevation can be drawn the height of the top edge must be found by obtaining the altitude of the triangle forming the end of the prism. All that is necessary, then, is to construct

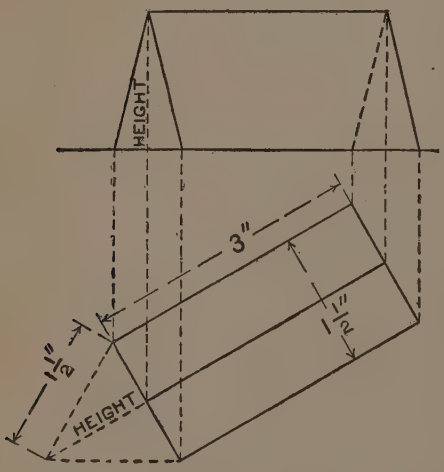

Fig 116. True Height on Horizontal Plane

an equilateral triangle $1\frac{1}{2}$ inches on each side, and measure its altitude. A very convenient way to do this is shown in the figure by laying one end of the prism down on H. A similar construction is shown in Fig. 117, but with one face of the prism on V instead of on H.

In all the work thus far the plan has been drawn below and the elevation above. This order is sometimes inverted and the plan put above the elevation, but the plan still remains a top view no matter where placed, so that after some practice it makes but little difference to the draughtsman which method is employed.

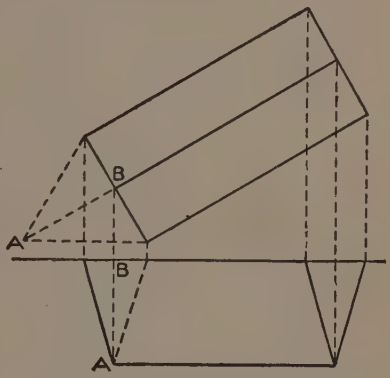

Fig. 117. True Height on Vertical Plane

ILLUSTRATIVE EXAMPLES

1. *Construct plan and elevation of a regular hexagonal pyramid.* It is evident that two distinct geometrical views are necessary to convey a complete idea of the form of the object; an elevation to represent the sides of the body, and to express its height; and a plan of the upper surface to express the form horizontally.

It is to be observed that this body has an imaginary axis or center-line, about which the same parts are equally distant; this is an essential characteristic of all symmetrical figures.

Draw a horizontal straight line $L\ T$ through the center of the sheet, Fig. 118; this line will represent the ground line. Then draw a perpendicular $Z\ Z'$ to the ground line. For the sake of preserving the symmetry of the drawing, the center of the lower range of figures are all in the same straight line $M\ N$, drawn parallel to the ground line.

In delineating the pyramid, it is necessary, in the first place, to construct the plan. The point S', where the line $Z\ Z'$ intersects the line $M\ N$, is to be taken as the center of the figure, and from this point, with a radius equal to the side of the hexagon which forms the base of the pyramid, describe a circle, cutting $M\ N$ in A' and D'. From these points with the same radius, draw four arcs of circles, cutting the primary circle in four points. These six points being

joined by straight lines, will form the figure $A'\ B'\ C'\ D'\ E'\ F'$, which is the base of the pyramid; and the lines $A'\ D'$, $B'\ E'$, and $C'\ F'$, will represent the projections of its edges foreshortened as they would appear in the plan. If this operation has been correctly performed, the opposite sides of the hexagon should be parallel to each other and to one of the diagonals; this should be tested by the application of the square or other instrument proper for the purpose.

By the help of the plan obtained as above described, the vertical projection of the pyramid may be easily constructed. Since its base rests upon the horizontal plane, it must be projected vertically upon

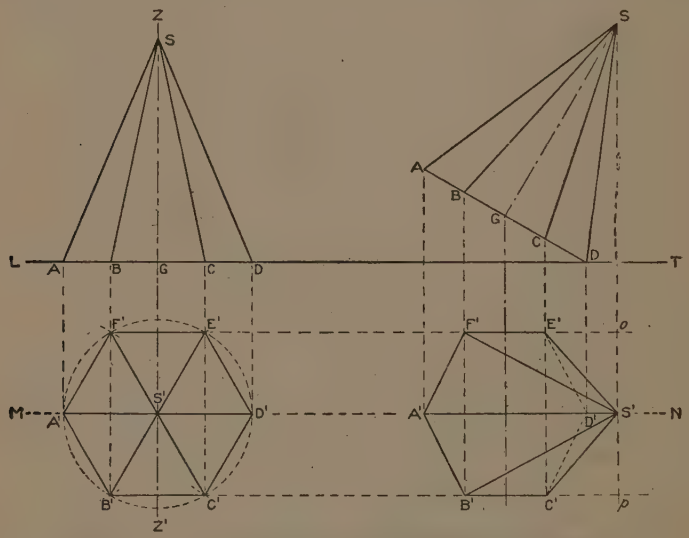

Fig. 118. Construction of Regular Hexagonal Pyramid

the ground line; therefore, from each of the points A', B', C', and D', raise perpendiculars to that line. The points of intersection, A, B, C, and D, are the true positions of all the angles of the base; and it only remains to determine the height of the pyramid, which is to be set off from the point G to S, and to draw $S\,A$, $S\,B$, $S\,C$, and $S\,D$, which are the only edges of the pyramid visible in the elevation. Of these it is to be remarked that $S\,A$ and $S\,D$ alone, being parallel to the vertical plane, are seen in their true length; and moreover, that from the assumed position of the solid under examination, the points F' and E' being situated in the lines $B\ B'$ and $C\ C'$, the lines $S\ B$ and $S\ C$ are each the projections of two edges of the pyramid.

2. *Construct the projections of the pyramid, Example 1, having its base set in an inclined position, but with its edges* S and $S\,D$ *still parallel to the vertical plane,* Fig. 118.

It is evident, that with the exception of the inclination, the vertical projection of this solid is precisely the same as in the preceding example, and it is only necessary to show the same view of the prism in its new position. For this purpose, after having fixed the position of the point D upon the ground line, draw through this point a straight line $D\,A$, making with $L\,T$ an angle equal to the desired inclination of the base of the pyramid. Then set off the distance $D\,A$, equal to that used in Example 1; erect a perpendicular on the center, and set off $G\,S$ equal to the height of the pyramid. Transfer also from the first example the distance $B\,G$ and $C\,G$ to the corresponding points, and complete the figure by drawing the straight lines $A\,S$, $B\,S$, $C\,S$, and $D\,S$.

In constructing the plan of the pyramid in this position, it is to be remarked, that since the edges $S\,A$ and $S\,D$ are still parallel to the vertical plane, and the point D remains unaltered, the projection of the points A, D, and S, will still be in the line $M\,N$. The position of A' is determined by the intersection of the perpendicular $A\,A'$ with $M\,N$. The remaining points, B', C', etc., in the projection of the base, are found in a similar manner, by the intersections let fall from the corresponding points in the elevation, with lines drawn parallel to $M\,N$, at a distance (set off at o, p,) equal to the width of the base. By joining all the contiguous points, the figure $A'\,B'\,C'\,D'\,E'\,F'$ is obtained representing the horizontal projection of the base, two of its sides, however, being dotted, as they must be supposed to be concealed by the body of the pyramid. The vertex S having been similarly projected to S', and joined by straight lines to the several angles of the base, the projection of the solid is completed.

INTERSECTIONS

If one surface meets another at some angle, an intersection is produced. Either surface may be plane, or curved. If both are plane, the intersection is a straight line; if one is curved, the intersection is a curve, except in a few special cases; and if both are curved, the intersection is usually curved. In the latter case, the entire curve

does not always lie in the same planes. If all points of any curve lie in the same plane, it is called a *plane curve.* A plane intersecting a curved surface must always give either a plane curve or a straight line.

Planes with Planes. In Fig. 119-a square pyramid is cut by a plane A parallel to the horizontal. This plane cuts from the pyramid a four-sided figure, the four corners of which will be the points where A cuts the four slanting edges of the solid. The plane intersects edge $o\,b$ at point 4^v in elevation. This point must be found in plan vertically below on the horizontal projection of line $o\,b$, that is, at point 4^h. Edge $o\,e$ is directly in front of $o\,b$, so is shown in elevation as the same line, and plane A intersects $o\,e$ at point 1^v in elevation, found in plan at 1^h. Points 3 and 2 are obtained in the same way. The intersection is shown in plan as the square 1-2-3-4, which is also its true size as it is parallel to the horizontal plane. In a similar way the intersections are found in Figs. 120 and 121. It will be seen that in these three cases where the planes are parallel to the bases, the sections are of the same shape as the bases, and have their sides parallel to the edges of the bases.

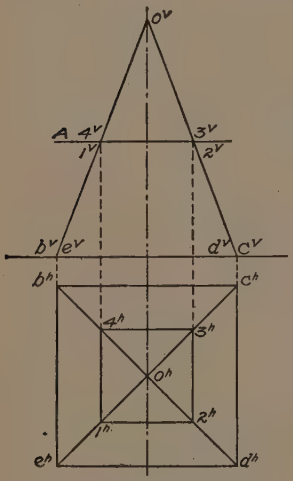

Fig. 119. Intersection of Plane and Pyramid

It is an invariable rule that when such a solid is cut by a plane parallel to its base, the section is a figure of the same shape as the base. If then in Fig. 122 a right cone is intersected by a plane parallel to the base the section must be a circle, the center of which in plan coincides with the apex. The radius must equal $o\,d$.

In Fig. 123 and Fig. 124 the cutting plane is not parallel to the base, hence the section will not be of the same shape as the base. The intersections are found, however, in exactly the same manner as in the previous figures, by projecting the points where the plane intersects the edges in elevation, on to the other view of the same line.

ILLUSTRATIVE EXAMPLES

1. Find the horizontal projection of a transverse section of the pyramid of Fig. 118, made by a plane perpendicular to the vertical,

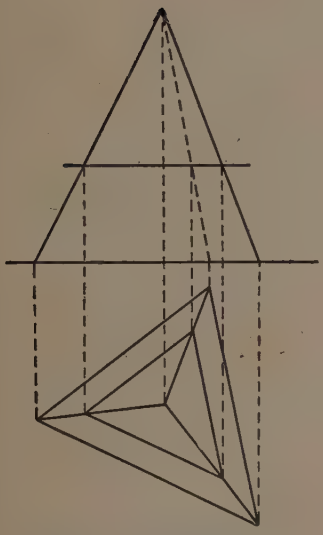

Fig. 120. Intersection of Plane and Pyramid

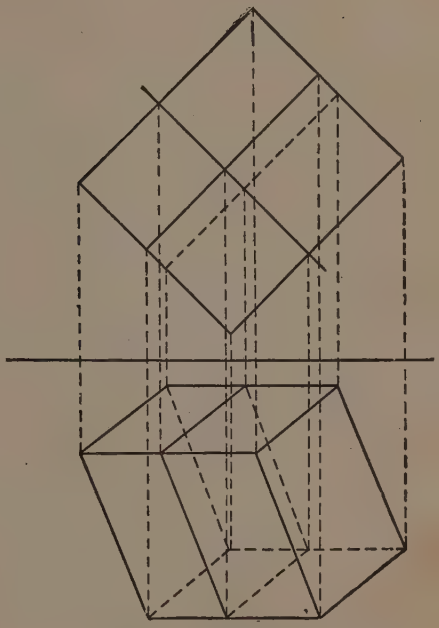

Fig. 121. Intersection of Plane and Prism

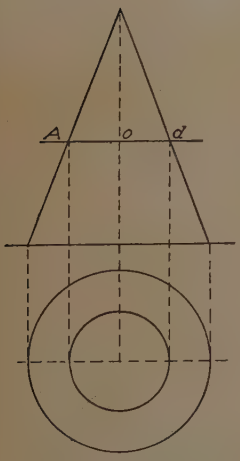

Fig. 122. Intersection of Plane and Cone

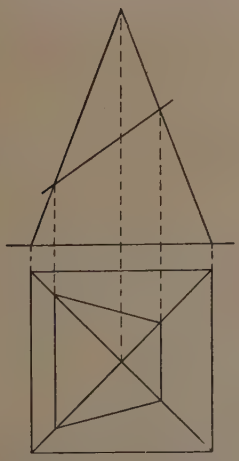

Fig. 123. Intersection of Plane and Square Pyramid

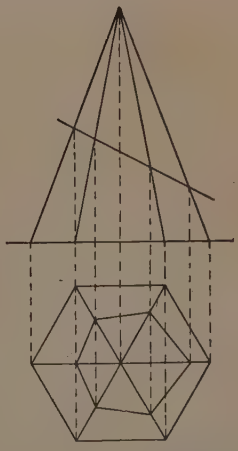

Fig. 124. Intersection of Plane and Hexagonal Pyramid

but inclined at an angle to the horizontal plane of projection; and let all the sides of the base be at an angle with the ground line, Fig. 125.

Having drawn the vertical $S\,S'$, the center line of the figures, its point of intersection with the line $M\,N$ is the center of the plan. Since none of the sides of the base are to be parallel with the ground line, draw a diameter $A'\,D'$ making the required angle with $M\,N$, and from the points A' and D' proceed to set out the angular points of the hexagon, as in Fig. 118. Then join the angular points which are diametrically opposite and project the figure thus obtained upon the vertical plane, as shown.

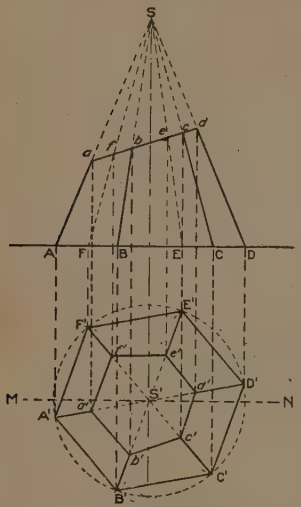

Fig. 125. Frustum of Hexagonal Pyramid

Now, if the cutting plane be represented by the line $a\,d$ in the elevation, it is obvious that it will expose, as the section of the pyramid, a polygon whose angular points being the intersections of the various edges with the cutting plane, will be projected in perpendiculars drawn from the points where it meets these edges respectively. From the points a, f, b, etc., let fall the perpendiculars aa', ff', bb', etc., to meet the lines $A'\,D'$, $F'\,C'$, $B'\,E'$, etc. When the contiguous points of intersection of these lines are joined, a six-sided figure will be formed which will represent the section required. The edges $F\,S$ and $E\,S$ being concealed in the elevation, but necessary for the construction of the plan, have been expressed in dotted lines, as is also the portion of the pyramid situated above the cutting plane which, though supposed to be removed, is necessary in order to draw the lines representing the edges.

2. Find the horizontal projection of the transverse section of a regular five-sided pyramid, cut by a plane perpendicular to the vertical, but inclined at an angle to the horizontal plane of projection; and let one edge of the pyramid be in a plane perpendicular to both the horizontal and the vertical planes of projection, as shown in Fig. 126.

The plan of the pyramid is constructed by describing from the center S', a circle circumscribing the base, and from B' dividing the circumference into five equal parts, and joining the contiguous points of division by straight lines. These form the polygon $A'\ B'\ C'\ D'\ E'$, whose angles, when joined to the center S', show the projections of the edges of the pyramid. Then, following the method above explained, the elevation and the horizontal projection of the section made by the plane $a\ c$ are obtained. But that method will not suffice for the determination of the point b', because the perpendicular let fall from the corresponding point b, in the elevation, coincides with the projection of the edge $B\ S$. Let the pyramid supposedly be turned a quarter of a revolution round its axis; the line $B'\ S'$ will then have assumed the position $S'\ b^2$. Project the point b^2 to b^3, and join Sb^3. Then, since the required point must also be conceived to have described a quarter of a circle in a plane parallel to the horizontal plane, and that its new position must be in the line Sb^3, it is obvious that its vertical projection is the point b^4, the intersection of a horizontal line drawn through b with the line Sb^3. The distance $b\ b^4$ may then be used to determine the distance from S' to b', and determines the position of the latter point in the plan; or, following a more methodical process, by projecting the point b^4 to b^5, and describing a circle from the center S' passing through b^5, its intersection with $B'\ S'$ is the point sought.

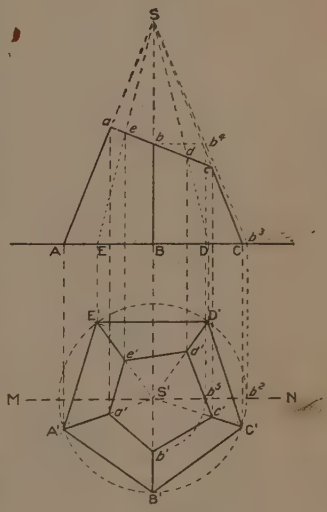

Fig. 126. Frustum of Pentagonal Pyramid

Planes with Cones or Cylinders. Sections cut by a plane from a cone have already been defined as *conic sections*. These sections may be any of the following: two straight lines, circle, ellipse, parabola, hyperbola. All except the parabola and hyperbola may also be cut from a cylinder.

Methods have previously been given for constructing the ellipse, parabola, and hyperbola, without projections; it will now be shown that they may be obtained as actual intersections.

Ellipse. In Fig. 127 the plane cuts the cone obliquely. To find points on the curve in plan take a series of horizontal planes x y z, etc., between points c^v and d^v. One of these planes, as w, should be taken through the center of c d. The points c and d must be points on the curve, since the plane cuts the two contour elements at these points. Contour elements are those forming the outline. The horizontal projections of the contour elements will be found

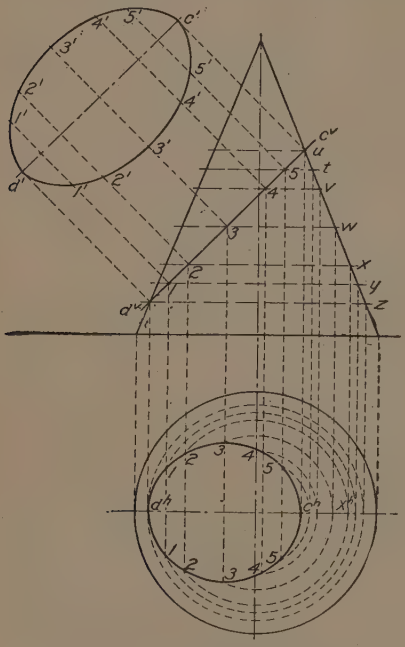

Fig. 127. Ellipse—Section from a Cone

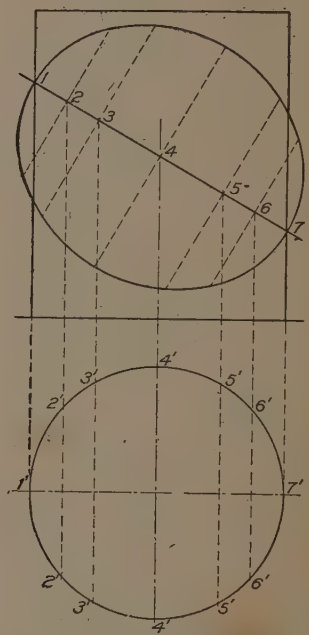

Fig. 128. Ellipse—Section from a Cylinder

in a horizontal line passing through the center of the base; hence the horizontal projection of c and d will be found on this center line, and will be the extreme ends of the curve.

The plane x cuts the surface of the cone in a circle, as it is parallel to the base, and the diameter of the circle is the distance between the points where x crosses the two contour elements. This circle, lettered x on the plan, has its center at the horizontal projection of the apex. The circle x and the curve cut by the plane are both on the surface of the cone, and their vertical projections intersect at the points 2-2. Point 2 on the elevation then represents two points

which are shown in plan directly beneath on the circle x, and are points on the required intersection. Planes y and z, and as many more as may be necessary to determine the curve accurately, are used in the same way. The curve found is an ellipse. The student will readily see that the true size of this ellipse is not shown in the plan, for the plane containing the curve is not parallel to the horizontal.

In order to find the *actual size of the ellipse*, it is necessary to

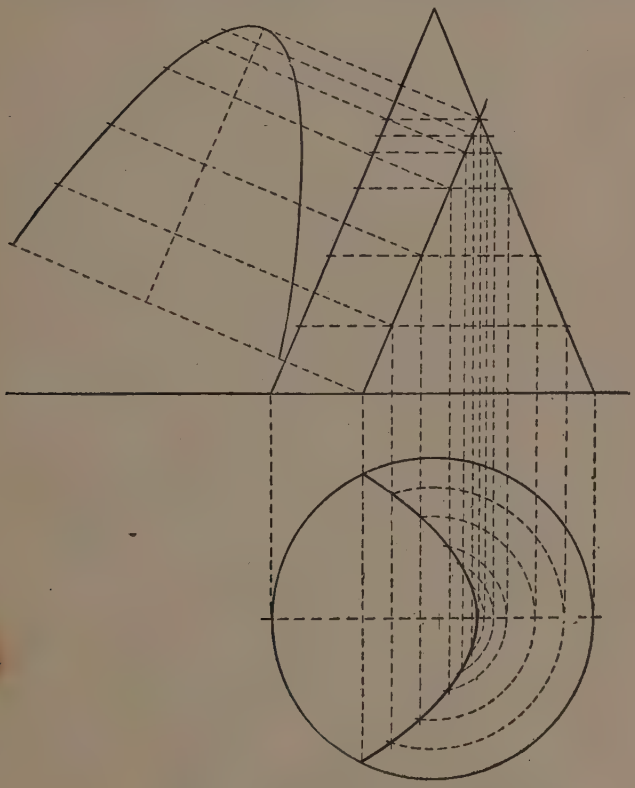

Fig. 129. Parabola—Section from a Cone

place its plane in a position parallel either to the vertical or to the horizontal. The actual length of the long diameter of the ellipse must be shown in elevation, $c^v\ d^v$, because the line is parallel to the vertical plane. The plane of the ellipse then may be revolved about $c^v\ d^v$ as an axis until it becomes parallel to V, when its true size will be shown. For the sake of clearness of construction, $c^v\ d^v$ is imagined moved over to the position $c'\ d'$, parallel to $c^v\ d^v$. The lines 1-1; 2-2;

3-3 on the plan show the true width of the ellipse, as these lines are parallel to H, but are projected closer together than their actual distances. In elevation these lines are shown as the points 1, 2, 3, at their true distance apart. Hence if the ellipse is revolved around its axis $c^v\ d^v$, the distances 1-1; 2-2; 3-3 may be laid off on lines perpendicular to $c^v\ d^v$, and the true size of the figure be shown.

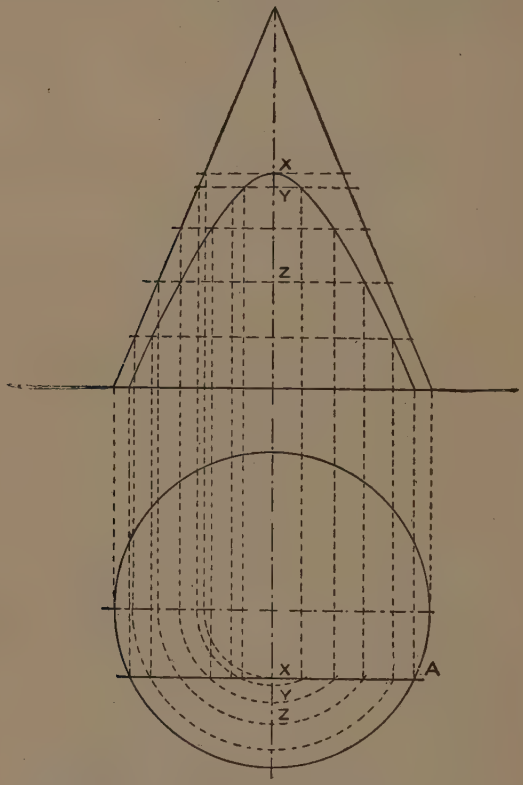

Fig. 130. Hyperbola—Section from a Cone

In Fig. 128 a plane cuts a cylinder obliquely. This is a simpler case, as the horizontal projection of the curve coincides with the base of the cylinder. To obtain the true size of the section, which is an ellipse, any number of points are assumed on the plan and projected up on the cutting plane, at 1, 2, 3, etc. The lines drawn through these points perpendicular to 1′-7′ are made equal in length to the corresponding distances 2′-2′; 3′-3′; etc., on the plan, because 2′-2′ is the true width of curve at 2.

Parabola. If a cone is intersected by a plane which is parallel to only one of the elements, as in Fig. 129, the resulting curve is the *parabola*, the construction of which is exactly similar to that for the ellipse as given in Fig. 127. If the intersecting plane is parallel to more than one element, or is parallel to the axis of the cone, a hyperbola is produced.

In Fig. 130, the vertical plane A is parallel to the axis of the cone. In this instance the curve when found will appear in its true size, as plane A is parallel to the vertical. Observe that the highest point of the curve is found by drawing the circle X on the plan tangent to the given plane. One of the points where this circle crosses the diameter is projected up to the contour element of the cone, and the horizontal plane X drawn. Intermediate planes Y, Z, etc., are chosen, and corresponding circles drawn in plan. The points where these circles are crossed by the plane A are points on the curve, and these points are projected up to the elevation on the planes Y, Z, etc.

DEVELOPMENT OF SURFACES

The development of a surface is the true size and shape of the surface extended or spread out on a plane. If the surface to be developed is of such a character that it may be flattened out without tearing or folding, an exact development is obtained, as in case of a cone or cylinder, prism or pyramid. If this cannot be done, as with the sphere, the development is only approximate.

Rectangular Prism. In order to find the development of the rectangular prism in Fig. 131, the back face, 1-2-7-6, is supposed to be placed in contact with some plane, then the prism turned on the edge 2-7 until the side 2-3-8-7 is in contact with the same plane, and this process continued until all four faces have been placed on the same plane. The rectangles 1-2-3-4 and 6-7-8-5 are for the top and bottom respectively. The development then is the exact size and shape of a covering for the prism. If a rectangular hole is cut through the prism, the openings in the front and back faces will be shown in the development in the centers of the two broad faces.

The development of a right prism, then, consists of as many rectangles joined together as the prism has sides, these rectangles being the exact size of the faces of the prism, and in addition two polygons the exact size of the bases. It will be found helpful in

developing a solid to number or letter all of the corners on the projections, then designate each face when developed in the same way as in the figure.

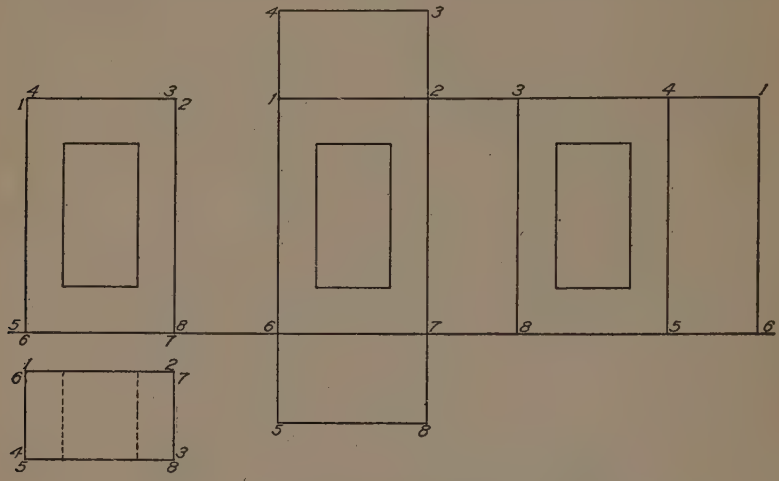

Fig. 131. Development of Hollow Rectangular Prism

Cone. If a cone be placed on its side on a plane surface, one element will rest on the surface. If now the cone be rolled on the plane, the vertex remaining stationary until the same element is in

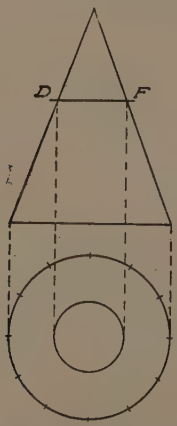

Fig. 132. Plan and Elevation of Cone

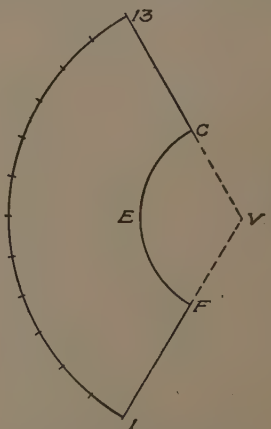

Fig. 133. Development of Cone

contact again, the space rolled over will represent the development of the convex surface of the cone. Fig. 132 is a cone cut by a plane parallel to the base. In Fig. 133, let the vertex of the cone be placed

at V, and one element of the cone coincide with $V\,F\,1$. The length of this element is taken from the elevation, Fig. 132, of either contour element. All of the elements of the cone are of the same length, so that when the cone is rolled, each point of the base as it touches the plane will be at the same distance from the vertex. From this it follows that in the development of the base, the circumference will become the arc of a circle of radius equal to the length of an element, and of a length equal to the distance around the base. To find this length divide the circumference of the base in the plan into any number of equal parts, say twelve, and lay off twelve such spaces, 1....13 along an arc drawn with radius equal to $V1$; join 1 and 13 with V, and the resulting sector is the development of the cone from vertex to base. In order to represent on the development the circle cut by the section plane $D\,F$, draw, from the vertex V as a center and with $V\,F$ as a radius, the arc $F\,C$. The development of the frustum of the cone will be a portion of a circular ring. This of course does not include the development of the bases, which would be simply two circles the same sizes as shown in plan.

Regular Triangular Pyramid. Fig. 134 represents the plan and elevation of a regular triangular pyramid, and Fig. 135, its develop-

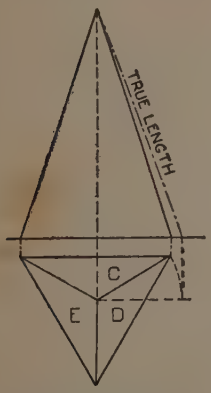

Fig. 134. Plan and Elevation of Triangular Pyramid

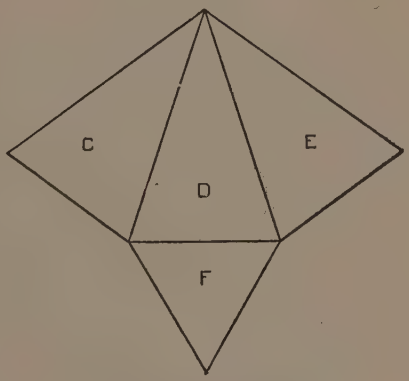

Fig. 135. Development of Triangular Pyramid

ment. If face C is placed on the plane its true size will be shown in the development. The true length of the base of triangle C is shown in the plan. As the slanting edges, however, are not parallel to the vertical, their true length is not shown in elevation but must

be obtained by the method given on Page 64, as indicated in Fig. 134. The triangle may now be drawn in its full size at *C* in the development, and as the pyramid is regular, two other equal triangles, *D* and *E*, may be drawn to represent the other sides. These, together with the base *F*, constitute the complete development.

Truncated Circular Cylinder. If a truncated circular cylinder is to be developed, or rolled upon a plane, the elements, being parallel, will appear as parallel lines, and the base line being perpendicular to the elements, will appear as a straight line of length equal to the circumference of the base. The base of the cylinder in Fig. 136 is

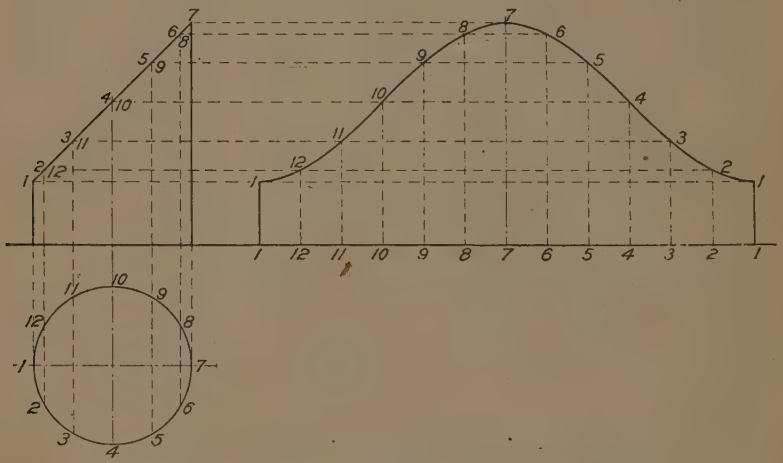

Fig. 136. Projections and Development of Truncated Cylinder

divided into twelve equal parts, 1, 2, 3, etc., and commencing at point 1 on the development, these twelve equal spaces are laid off along the straight line, giving the total width.

Draw in elevation the elements corresponding to the various divisions of the base, and note the points where they intersect the oblique plane. As the cylinder is rolled beginning at point 1, the successive elements, 1, 12, 11, etc., will appear at equal distances apart, and equal in length to the lengths of the same elements in elevation. Thus point number 10 on the development is found by projecting horizontally across from 10 in elevation. It will be seen that the curve formed is symmetrical, the half on the left of 7 being similar to that on the right. The development of any similar surface may be found in the same manner.

The principle of cylinder development is used in laying out elbow joints, pipe ends cut off obliquely, etc. In Fig. 137 is shown plan and elevation of a three-piece elbow and collar, and developments of the four pieces. In order to construct the various parts making up the joint, it is necessary to know what shape and size must be marked out on the flat sheet metal so that when cut out and rolled up the three pieces will form cylinders with the ends

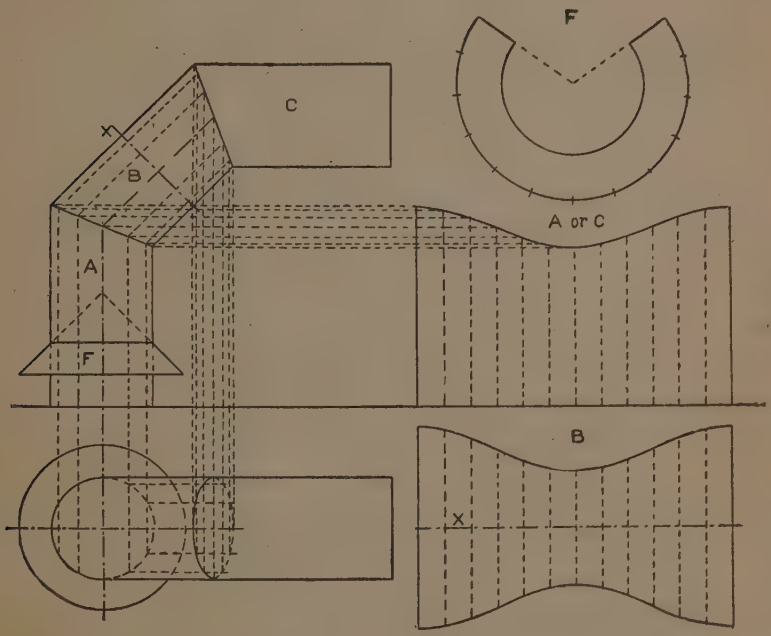

Fig. 137. Plan, Elevation, and Development of Three-Piece Elbow and Collar

fitting together as required. Knowing the kind of elbow desired, first draw the plan and elevation, and from these make the developments. Let the lengths of the three pieces *A*, *B*, and *C* be the same on the upper outside contour of the elbow, the piece *B* at an angle of 45°; the joint between *A* and *B* bisects the angle between the two lengths, and in the same way the joint between *B* and *C*. The lengths *A* and *C* will then be the same, and one pattern will answer for both. The development of *A* is made exactly as just explained for Fig. 136, and this is also the development of *C*.

It should be borne in mind that in developing a cylinder the

base must always be at right angles to the elements, and if the cylinder as given does not have such a base, it becomes necessary to cut the cylinder by a plane perpendicular to the elements, and use the intersection as a base. This point must be clearly understood in order to proceed intelligently. A section at right angles to the elements is the only section which will unroll in a straight line, and is, therefore, the section from which the other sections must be developed. As *B*, Fig. 137, has neither end at right angles to its length, the plane *X* is drawn at the middle and perpendicular to the length. *B* has the same diameter as *C* and *A*, so the section cut by *X* will be a circle of the same diameter as the base of *A*, and is shown in the development at *X*.

The elements on *B* are drawn from the points where the elements on the elevation of *A* meet the joint between *A* and *B*, and are equally spaced as shown on the plan of *A*. Commencing with the left-hand element in *B*, the length of the upper element between *X* and the top corner of the elbow is laid off above *X*, giving the first point in the development of the end of *B* fitting with *C*. The lengths of the other elements in the elevation of *B* are measured in the same way and laid off from *X*. The development of the other end of the piece *B* is laid off below *X*, using the same distances, since *X* is half way between the ends. The development of the collar is simply the development of the frustum of a cone, which has already been explained, Fig. 133. The joint between *B* and *C* is shown in plan as an ellipse, the construction of which the student should be able to understand from a study of the figure.

Intersection of Rectangular Prism and Pyramid. The intersection of a rectangular prism and a pyramid is shown in Fig. 138. The base *b c d e* of the pyramid is shown dotted in plan, as it is hidden by the prism. All four edges of the pyramid pass through the top of the prism 1-2-3-4. As the top of the prism is a horizontal plane, the edges of the pyramid are shown passing through the top in elevation at x^v g^v i^v k^v. These four points might be projected to the plan on the four edges of the pyramid; but it is unnecessary to project more than one, since the general principle applies here that if a cone, pyramid, prism or cylinder be cut by a plane parallel to the base, the section is a figure parallel and similar to the base. The one point x^v is therefore projected down to *a b* in plan, giving

x^h, and with this as one corner, the square $x^h\ g^h\ i^h\ k^h$ is drawn, its sides parallel to the edges of the base. This square is the intersection of the pyramid with the top of the prism.

The intersection of the pyramid with the bottom of the prism is found in like manner, by taking the point where one edge of the pyramid as $a\ b$ passes through the bottom of the prism shown in elevation as point m^v, projecting down to m^h on $a^h\ b^h$, and drawing

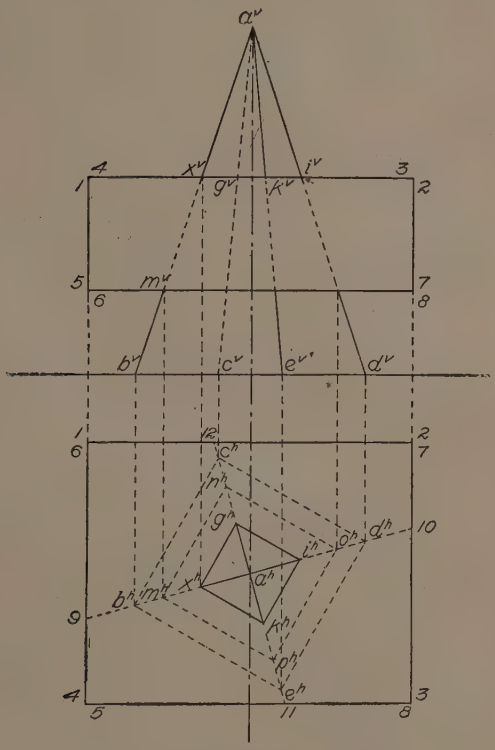

Fig. 138. Intersection of Prism and Pyramid

the square $m^h\ n^h\ o^h\ p^h$ parallel to the base of the pyramid. These two squares constitute the entire intersection of the two solids, the pyramid going through the bottom and coming out at the top of the prism. As much of the slanting edges of the pyramid as are above the prism will be seen in plan, appearing as the diagonals of the small square, and the rest of the pyramid, being below the top surface of the prism, will be dotted in plan.

Fig. 139 is the development of the rectangular prism, showing the openings in the top and bottom surfaces through which the pyramid passed. The development of the top and the bottom, the back and the front faces will be four rectangles joined together, the same sizes as the respective faces. Commencing with the bottom face 5-6-7-8, next would come the back face 6-1-2-7, then the top, etc. The rectangles at the ends of the top face 1-2-3-4 are the ends of the prism. These might have been joined on any other face as well. Now find the development of the square in the bottom 5-6-7-8. As the size will be the same as in projection, it only remains

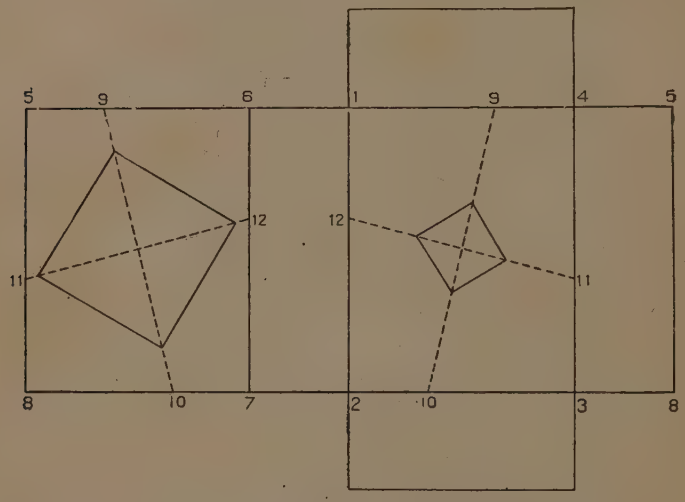

Fig. 139. Development of Objects Shown in Fig. 138

to determine its position. This position, however, will have the same relation to the sides of the rectangle as in the plan. The center of the square in this case is in the center of the face. To transfer the diagonals of the square to the development, extend them in plan, Fig. 138, to intersect the edges of the prism in points 9, 10, 11, and 12. Take the distance from 5 to 9 along the edge 5-6, and lay it on the development, Fig. 139, from 5 along 5-6, giving point 9. Point 10, located in the same way and connected with 9, gives the position of one diagonal. The other diagonal is obtained in a similar way, and the square is constructed on these diagonals. The same method is used for locating the small square on the top face.

Intersection of Triangular Prism and Cylinder. The intersection of a cylinder and prism is found by obtaining the points where elements of the cylinder pierce the prism, or where edges and lines parallel to edges on the surface of the prism cut the cylinder.

A series of parallel planes may also be taken cutting curves from the cylinder and straight lines from the prism; the intersections give points on the intersection of the two solids.

Fig. 140 represents a triangular prism intersecting a cylinder. The axis of the prism is parallel to V and inclined to H. Starting

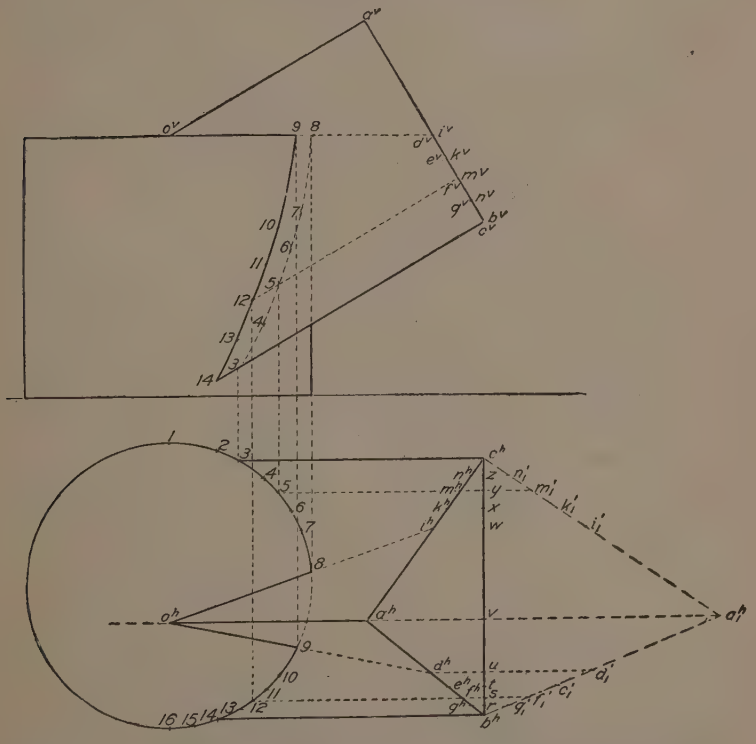

Fig. 140. Plan and Elevation of Intersection of Cylinder and Triangular Prism

with the size and shape of the base, this is laid off at $a_1^h\ b^h\ c^h$, and the altitude of the triangle is taken and laid off at $a^v\ c^v$ in elevation, making right angles with the inclination of the axis to H. The plan of the prism is then constructed. To find the intersection of the two solids, lines are drawn on the surface of the prism parallel

to the length and the points where these lines and the edges pierce the cylinder are obtained and joined, giving the curve.

The top edge of the prism goes into the top of the cylinder. This point will be shown in elevation, since the top of the cylinder is a plane parallel to H and perpendicular to V, and therefore projected on V as a straight line. The upper edge, then, is found to pass into the top of the cylinder at point o^v and o^h. The intersection of the two upper faces of the prism with the top of the cylinder will be straight lines drawn from point o and will be shown in plan. If a point can be found where another line of the surface $o\ a\ b$ 14 pierces the upper base of the cylinder, this point joined with o will determine the intersection of this face with the top of the cylinder. A surface may always be produced, if necessary, to find an intersection.

Edge $a\ b$ pierces the plane of the top of the cylinder at point d, shown in the elevation; therefore the line joining this point with o is the intersection of one upper face of the prism with the upper base of the cylinder. The only part of this line needed, of course, is within the actual limits of the base, that is, o 9. The intersection o 8 of the other top face is found by the same method. On the convex surface of the cylinder there will be three curves, one for each face of the prism. Points 8 and 9 on the upper base of the cylinder, will be where the curves for the two upper faces will begin. The point d is found on the revolved position of the base at d_1, and $d_1\ b$ is divided into the equal parts d_1 - e_1, e_1 - f_1, etc., which revolve back to d^h, e^h, f^h, and g^h. The divisions are made equal merely for convenience in developing. The vertical projections of d, e, etc., are found on the vertical projection of $a\ b$, directly above d^h, e^h, etc., or may be found by taking from the revolved position of the base, the perpendiculars from $d_1\ e_1$, etc., to $c^h\ b^h$ and laying them off in elevation from b^v along $b^v\ a^v$. Lines such as f 12, m 5, etc., parallel to $a\ o$ are drawn in plan and elevation. Points i^h, k^h, m^h, n^h are taken directly behind d^h, e^h, f^h, g^h, hence their vertical projections coincide. Points n_1, m_1, k_1, and i_1 are formed by projecting across from n^h, m^h, k^h, and i^h.

The convex surface of the cylinder is perpendicular to H, so the points where the lines on the prism pierce it will be projected on plan as the points where these lines cross the circle, 14, 13, 12,

11....3. The vertical projections of these points are found on the corresponding lines in elevation, and the curves drawn through. The curve 3, 4...8 must be dotted, as it is on the back of the cylinder. The under face of the prism, which ends with the line *b c*, is perpendicular to the vertical plane, so the curve of intersection will be projected on *V* as a straight line. Point 14 is one end of this curve, 3 the other end, and the curve is projected in elevation as the straight line from 14 to the point where the lower edge of the prism crosses the contour element of the cylinder.

Fig. 141 gives the development of the right-hand half of the cylinder, beginning with number 1. As previously explained, the distance between the elements is shown in the plan, as 1-2, 2-3, 3-4,

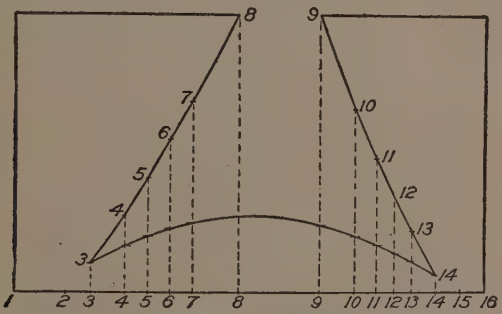

Fig. 141. Development of Right-Hand Half of Cylinder Shown in Fig. 140

and so on. These spaces are laid off in the development along a straight line representing the development of the base, and from these points the elements are drawn perpendicularly.

The lengths of the elements in the development from the base to the curve are exactly the same as on the elevation, as the elevation gives the true lengths. If then the development of the base is laid off along the same straight line as the vertical projection of the base, the points in elevation may be projected across with the T-square to the corresponding elements in the development. The points on the curve cut by the under face of the prism are on the same elements as the other curves, and their vertical projections are on the under edge of the prism, hence these points are projected across to obtain the lower curve.

In Fig. 142 is given the development of the prism from its right

hand end to its intersection with the cylinder, beginning at the left with the top edge *a o*, the straight line *a b c a* being the base in the development. As this must be the actual distance around the base, the length is taken from the true size of the base $a_1^h\ b^h\ c^h$, in Fig. 140.

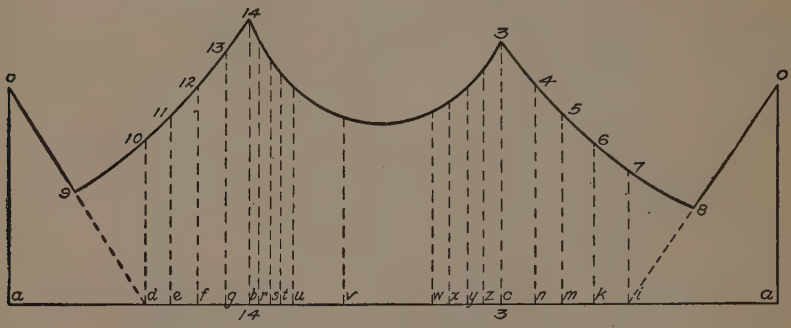

Fig. 142. Development of Prism Shown in Fig. 140

The parallel lines drawn on the surfaces of the prism must appear on the development their true distances apart, hence the distances $a_1^h\ d_1'$, $d_1'\ e_1'$, etc., are made equal to *a d*, *d e*, etc., on the development. The actual distances between the parallel lines on the bottom face of the prism are shown along the edge of the base $b^h\ c^h$ in Fig. 140. Perpendicular lines are drawn from the points of division on the development.

The position of the curve is found by laying off the true lengths on the perpendiculars. These true lengths (of the parallel lines) are not shown in plan, as the lines are not parallel to the horizontal plane, but are found in elevation. The length *o a* on the development is equal to $a^v\ o^v$, the length *d* 10 is equal to d^v 10, and so on for all the rest. Point 9 is found as follows: on the projections, the straight line from *o* to *d* passes through point 9, and the true distance from *o* to 9 is shown in plan. All that is necessary, then, is to connect *o* and *d* on the development, and lay off from *o* the distance $c^h 9$. Point 8 is found in the same way.

ISOMETRIC PROJECTION

In orthographic projection an object has been represented by two or more projections; another system, called *isometrical drawing*, is often used to show in one view the three dimensions of an object,

length (or height), breadth, and thickness. An isometrical drawing of an object, as a cube, is called for brevity the *isometric* of the cube.

To obtain a view which shows the three dimensions in such a way that measurements may be taken from them, draw the cube in the simple position shown at the left, Fig. 143, in which it rests on H with two faces parallel to V; the diagonal from the front upper right-hand corner to the back lower left-hand corner is indicated by the dotted line. Swing the cube around until the diagonal is parallel with V as shown in the second position. Here the front face is at the right. In the third position the lower end of the diagonal has been raised so that it is parallel to H, becoming thus parallel to both planes. The plan is found by the principles of projection, from the elevation and the preceding plan. The front face is now the lower of the two faces shown in the elevation. From this position the cube is swung around, using the corner resting on H as a pivot, until the diagonal is perpendicular to V but still parallel to H. The plan remains the same, except as regards position; while the elevation, obtained by projecting across from the previous elevation, gives the isometrical projection of the cube. The front face is now at the left.

In the last position, as one diagonal is perpendicular to V, it follows that all the faces of the cube make equal angles with V, hence

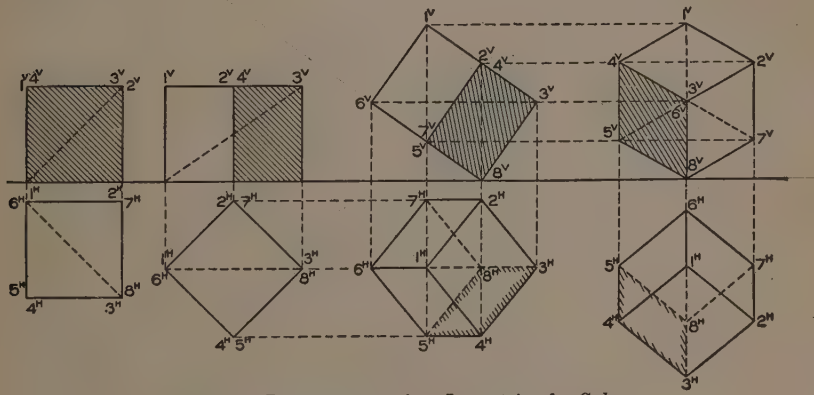

Fig. 143. Development of an Isometric of a Cube

are projected on that plane as equal parallelograms. For the same reason all the edges of the cube are projected in elevation in equal lengths, but, being inclined to V, appear shorter than they actually are on the object. Since they are all equally foreshortened and since

a drawing may be made at any scale, it is customary to make all the isometrical lines of a drawing full length. This will give the same proportions, and is much the simpler method. Herein lies the distinction between an isometric projection and an isometric drawing.

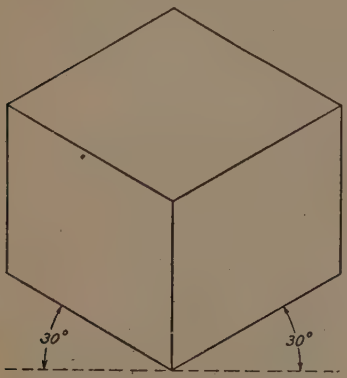

Fig. 144. Isometric of a Cube

It will be noticed that the figure may be inscribed in a circle, and that the outline is a perfect hexagon. Hence the lines showing breadth and length are 30° lines, while those showing height are vertical.

Fig. 144 shows the isometric of a cube 1 inch square. All of the edges are shown in their true length, hence all the surfaces appear of the same size. In the figure the edges of the base are inclined at 30° with a T-square line, but this is not always the case. For rectangular objects, such as prisms,

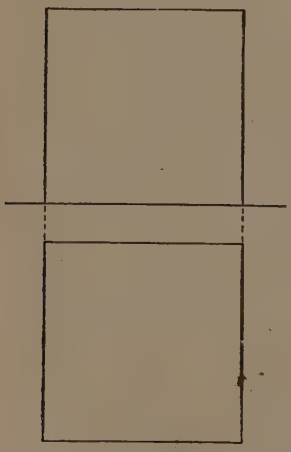
Fig. 145. Plan and Elevation of a Cube

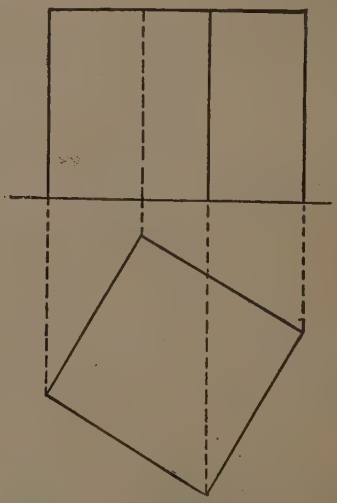
Fig. 146. Cube of Fig. 145 Rotated 30° with Vertical Plane

cubes, etc., the base edges are at 30° only when the prism or cube is supposed to be in the simplest possible position. The cube in Fig. 144 is supposed to be in the position indicated by plan and

elevation in Fig. 145, that is, standing on its base, with two faces parallel to the vertical plane.

If the isometric of the cube in the position shown in Fig. 146 were required, it could not be drawn with the base edges at 30°; neither would these edges appear in their true lengths. It follows, then, that in isometrical drawing, true lengths appear only as 30° lines or as vertical lines. Edges or lines that in actual projection are either parallel to the ground line or perpendicular to V, are drawn in isometric as 30° lines, full length; and those that are actually vertical are made vertical in isometric, also full length.

In Fig. 143, lines such as the front vertical edges of the cube and the two base edges are called the *three isometric axes*. The isometric of objects in oblique positions, as in Fig. 146, can be constructed only by reference to their projections, by methods which will be explained later.

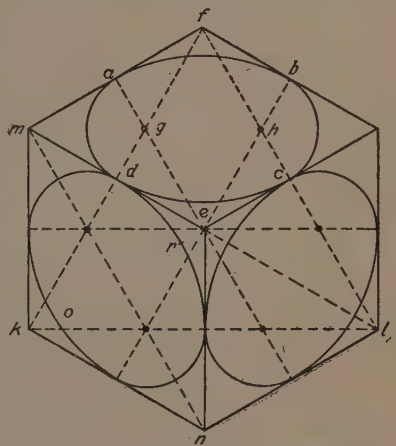

Fig. 147. Isometric of a Cube with Circles Inscribed on its Faces

In isometric drawing small rectangular objects are more satisfactorily represented than large curved ones. In woodwork, mortises and joints and various parts of framing are well shown in isometric. This system is used also to give a kind of bird's-eye view of mills or factories. It is also used in making sketches of small rectangular pieces of machinery, where it is desirable to give shape and dimensions in one view.

Fig. 147 shows a cube with circles inscribed in the top and two side faces. The isometric of a circle is an ellipse, the exact construction of which would necessitate finding a number of points; for this reason an approximate construction by arcs of circles is often made. In the method, Fig. 147, four centers are used. Considering the upper face of the cube, lines are drawn from the obtuse angles f and e, to the centers of the opposite sides. The intersections of these lines give points g and h, which serve as centers for the ends of the ellipse. With g as center and radius $g\,a$, the arc

$a\ d$ is drawn, and with f as center and radius $f\ d$, the arc $d\ c$ is drawn; the ellipse is finished by using centers h and e. This construction is applied to all three faces.

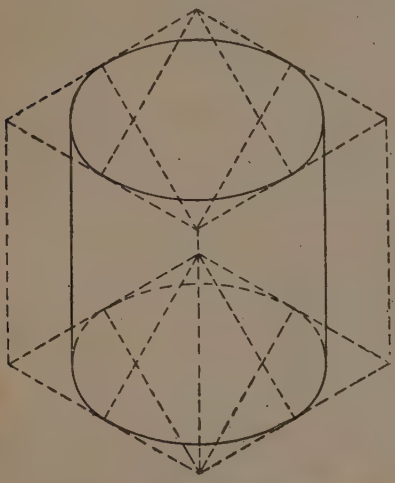

Fig. 148. Isometric of a Cylinder

Fig. 148 is the isometric of a cylinder standing on its base.

Fig. 149 represents a block with smaller blocks projecting from three faces.

Fig. 150 shows a framework of three pieces, two at right angles and a slanting brace. The horizontal piece is mortised into the upright, as indicated by the dotted lines.

In Fig. 151 the isometric outline of a house is represented, showing a dormer window and a partial hip roof; $a\ b$ is a hip rafter, $c\ d$ a valley. Let the pitch of the main roof be shown at B, and let m be the middle point of the top of the end wall of the house. Then, by measuring vertically up a distance $m\ l$ equal to the vertical height

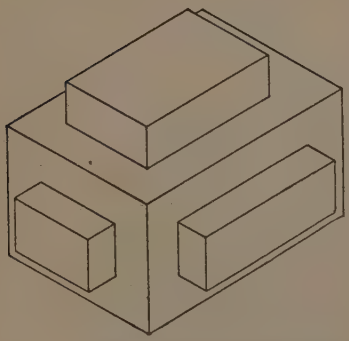

Fig. 149. Isometric of a Wooden Block

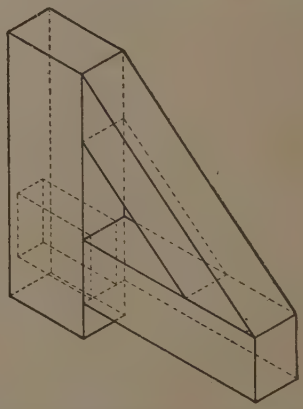

Fig. 150. Isometric of a Wooden Brace

$a\ n$ shown at B, a point on the line of the ridge will be found at l. Line $l\ i$ is equal to $b\ h$, and $i\ h$ is then drawn. Let the pitch of the end roof be given at A. This shows that the peak of the roof,

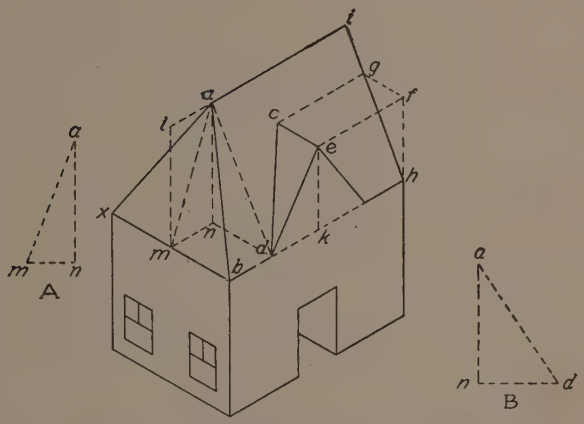

Fig. 151. Isometric Outline of a House

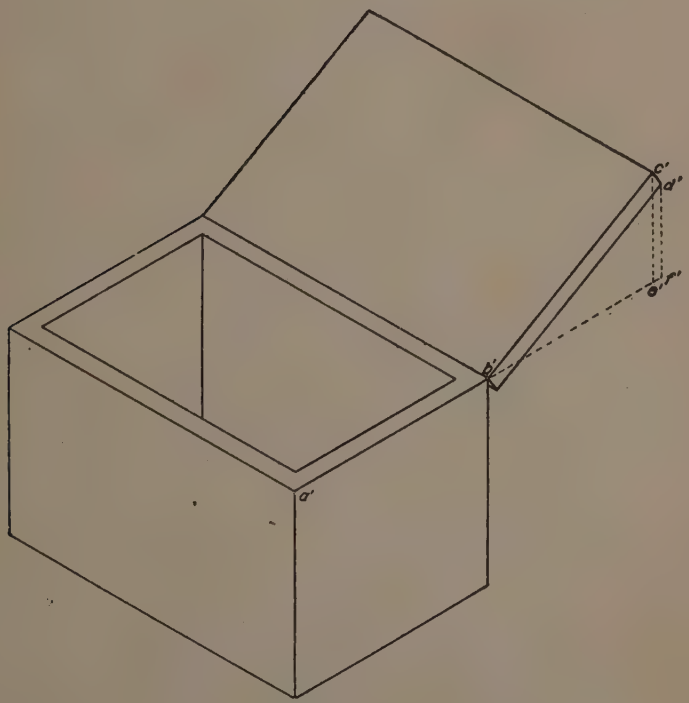

Fig. 152. Isometric of a Box and Cover

or the end a of the ridge, will be back from the end wall a distance equal to the base of the triangle at A. Hence, lay off from l this distance, giving point a, and join a with b and x.

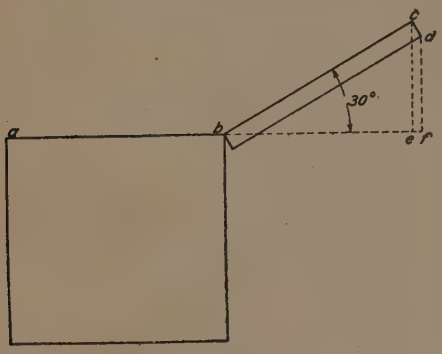

Fig. 153. End View of Box Shown in Fig. 152

The height $k\,e$ of the ridge of the dormer roof is known, and it must be found where this ridge will meet the main roof. The ridge must be a 30° line as it runs parallel to the end wall of the house and to the ground. Draw from e a line parallel to $b\,h$ to meet a vertical through h and f. This point is in the vertical plane of the end wall of the house, hence in the plane of $i\,h$. If now a 30° line be drawn from f parallel to $x\,b$, it will meet the roof of the house at g. The dormer ridge and $f\,g$ are in the same horizontal plane, hence will meet the roof at the same distance below the ridge $a\,i$. Therefore draw the 30° line $g\,c$, and connect c with d.

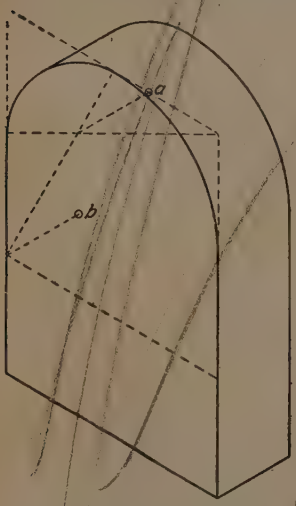

Fig. 154. Isometric of a Prism with Semicircular Top

In Fig. 152 a box is shown with the cover opened through 150°. The right-hand edge of the bottom shows the width, the left-hand edge shows the length and the vertical edge shows the height. The short edges of the cover are not isometric lines, hence are not shown in their true lengths; neither is the angle through which the cover is opened represented in its actual size.

The corners of the cover must then be determined by co-ordinates from an end view of the box and cover. As the end of the cover is in the same plane as the end of the box, the simple end view as shown in Fig. 153 will be sufficient. Extend the top of the box to the right, and from c and d let fall perpendiculars on $a\,b$ produced, giving the points e and f. The point c may be located by means

of the two distances or co-ordinates $b\ e$ and $e\ c$, and these distances will appear in their true lengths in the isometric view. Hence pro-

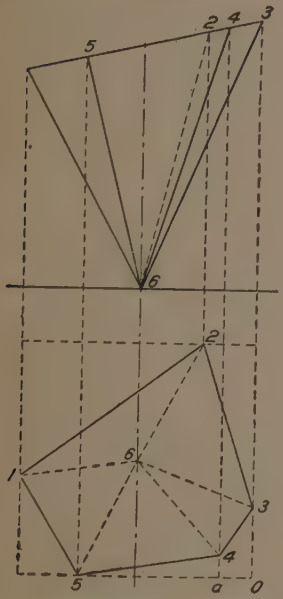

Fig. 155. Plan and Elevation of Oblique Pentagonal Pyramid

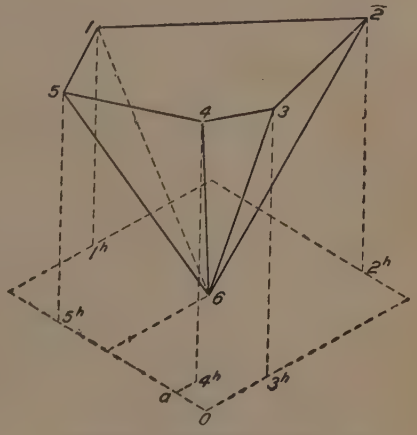

Fig. 156. Isometric of Fig. 155

duce $a'\ b'$ to e' and f'; and from these points draw verticals $e'\ c'$ and $f'\ d'$; make $b'\ e'$ equal to $b\ e$, $e'\ c'$ equal to $e\ c$; and similarly for d'. Draw the lower edge parallel to $c'\ d'$ and equal to it in length, and connect with b'.

It will be seen that in isometric drawing parallel lines always appear parallel. It is also true that lines divided proportionally maintain this same relation in isometric drawing.

Fig. 154 shows a block or prism with a semicircular top. Find the isometric of the square circumscribing the circle, then draw the curve by the approxi-

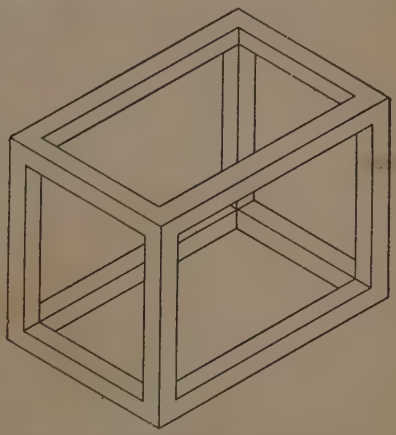

Fig. 157. Isometric of a Skeleton of a Box

mate method. The centers for the back face are found by projecting the front centers back 30° equal to the thickness of the

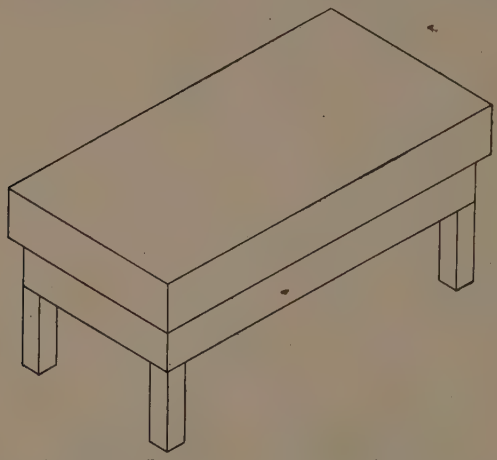

Fig. 158. Isometric of a Carpenter's Bench

prism, as shown at *a* and *b*. The plan and elevation of an oblique pentagonal pyramid are shown in Fig. 155. It is evident that none of the edges of the pyramid can be drawn in isometric

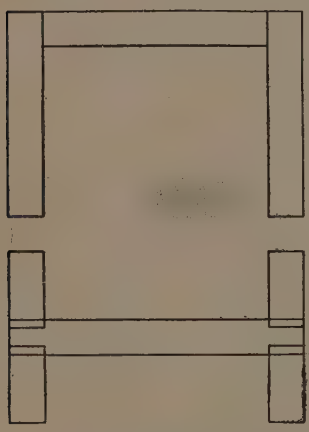

Fig. 159. Plan and Elevation of Sawhorse

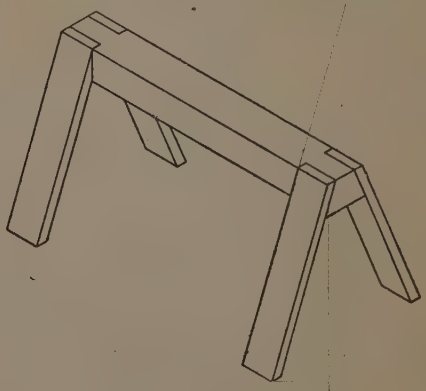

Fig. 160. Isometric of Fig. 159

as either vertical or 30° lines; hence, a system of co-ordinates must be used as shown in Fig. 156. This problem illustrates the most general case; and to locate some of the points three co-ordinates must be used, two at 30° and one vertical.

Circumscribe, about the plan of the pyramid, a rectangle which shall have its sides respectively parallel and perpendicular to the

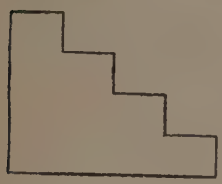

Fig. 161. End Elevation of Stairs

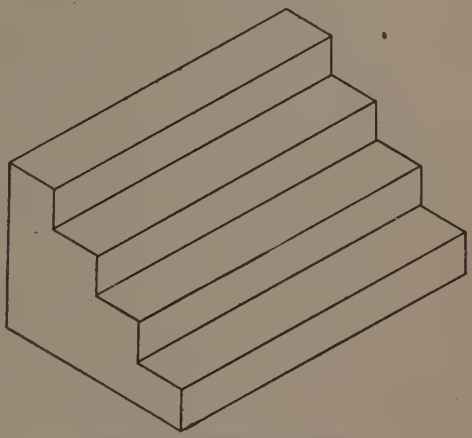

Fig. 162. Isometric of Stairs

ground line. This rectangle is on H, and its vertical projection is in the ground line.

The isometric of this rectangle can be drawn at once with 30° lines, as shown in Fig. 156, o being the same point in both figures. The horizontal projection of point 3 is found in isometric at 3^h, at the same distance from o as in the plan. That is,

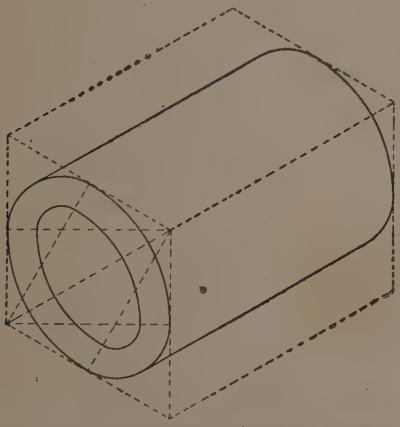

Fig. 163. Isometric of a Hollow Cylinder

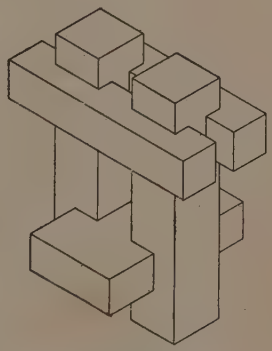

Fig. 164. Isometric of a Wooden Model

any distance which in plan is parallel to a side of the circumscribing rectangle, is shown in isometric in its true length and parallel to the corresponding side of the isometric rectangle. If point 3 were on the horizontal plane its isometric would be 3^h, but the point is at the vertical height above H given in the elevation; hence, lay off above

3^h this vertical height, obtaining the actual isometric of the point. To locate point 4, draw 4 *a* parallel to the side of the rectangle; then lay off *o a* and *a* 4^h, giving what may be called the isometric plan of 4. The vertical height taken from the elevation locates the isometric of the point.

In like manner all the corners of the pyramid, including the apex, are located. The rule is, *locate first in isometric the horizontal*

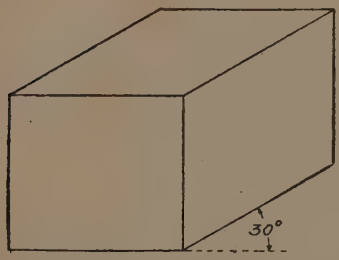

Fig. 165. Oblique View of Cube at 30°

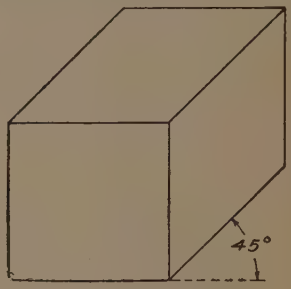

Fig. 166. Oblique View of Cube at 45°

projection of a point by one or two 30° *co-ordinates; then vertically above this point, locate its height as taken from the elevation.*

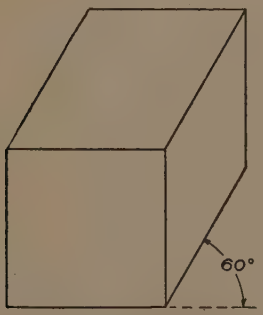

Fig 167. Oblique View of Cube at 60°

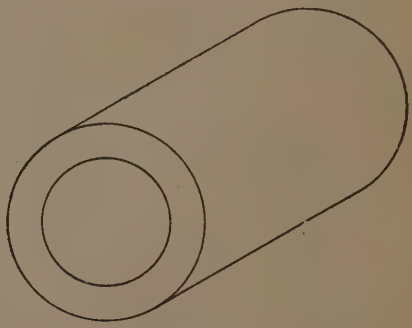

Fig. 168. Oblique View of Hollow Cylinder

Figs. 157 to 164 give examples of the isometric of various objects. Fig. 159 is the plan and elevation, and Fig. 160 the isometric, of a carpenter's sawhorse.

OBLIQUE PROJECTIONS

In oblique projection, as in isometric, the end sought for is the same—a more or less complete representation, in one view, of

any object. Oblique projection differs from isometric in that one face of the object is represented as if parallel to the vertical plane of projection, the others inclined to it. Another point of difference is that oblique projection cannot be deduced from orthographic projection, as is isometric.

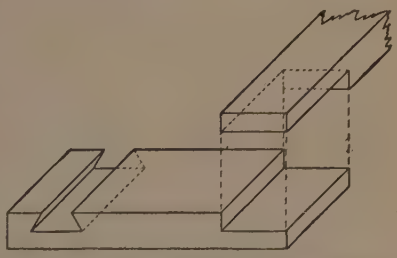

Fig. 169. Oblique View of a Mitre Joint

In oblique projection all lines in the front face are shown in their true lengths and in their true relation to one another, and lines which are perpendicular to this front face are shown in their true lengths at any angle that may be desired for any particular case. Lines not in the plane of the front face nor perpendicular to it must be determined by co-ordinates, as in isometric. It will be seen at once that this system possesses some advantages over the isometric, as, for instance, in the representation of circles, as any circle or curve in the front face is actually drawn as such. Fig. 165, Fig. 166, and Fig. 167 show a cube in oblique projection with the 30°, 45°, and 60° slant, respectively. Fig. 168 shows a hollow cylinder in oblique projection. Figs. 169, 170, 171, 173 are other examples of oblique projections. Fig. 171 is a crank arm.

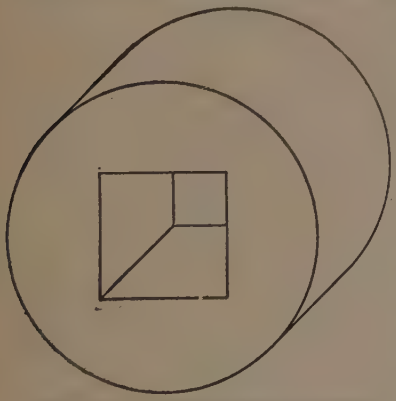

Fig. 170. Oblique View of Cylinder

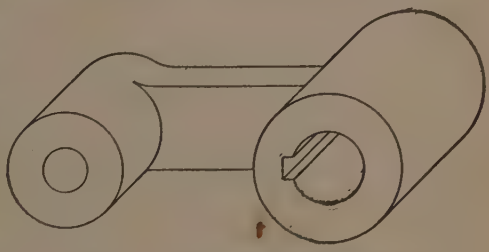

Fig. 171. Oblique View of Crank Shaft

The method of using co-ordinates for lines of which the true lengths are not shown, is illustrated by Figs. 172 and 173. Fig. 173 represents the oblique projection of the two joists shown in plan

and elevation in Fig. 172. The dotted lines in the elevation, Fig. 172, show the heights of the corners above the horizontal stick. The feet of these perpendiculars give the horizontal distances of the top corners from the end of the horizontal piece.

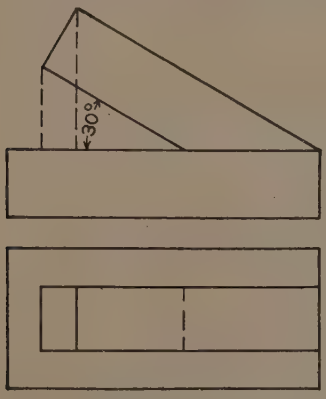

Fig. 172. Plan and Elevation of Wooden Brace

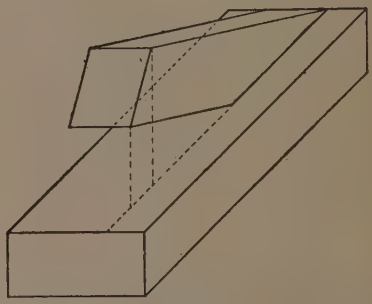

Fig. 173. Oblique View of Wooden Brace

In Fig. 173 lay off from the upper right-hand corner of the front end a distance equal to the distance between the front edge of the inclined piece and the front edge of the bottom piece, Fig. 172. From this point draw a dotted line parallel to the length. The horizontal distances from the upper left corner to the dotted perpendicular are then marked off on this line. From these points verticals are drawn, and made equal in length to the dotted perpendiculars of Fig. 172, thus locating two corners of the end.

LINE SHADING

In finely finished drawings it is frequently desirable to make the various parts more readily seen by showing the graduations of light and shade on the curved surfaces. This is especially true of such surfaces as cylinders, cones, and spheres. The effect is obtained by drawing a series of parallel or converging lines on the surface at varying distances from one another. Sometimes draftsmen vary the width of the lines, themselves. These lines are farther apart on the lighter portion of the surface, and closer together and heavier on the darker part.

Fig. 174 shows a cylinder with elements drawn on the surface

equally spaced, as on the plan. On account of the curvature of the surface the elements are not equally spaced on the elevation,

Fig. 174. Plan and Shaded Elevation of Cylinder

Fig. 176. Shaded Horizontal Cylinder

Fig. 175. Shaded Vertical Cylinder

Fig. 177. Shaded Section of Hollow Cylinder

but give the effect of graduation of light. The result is that in elevation the distances between the elements gradually lessen from the

Fig. 178. Shaded Elbow Joint

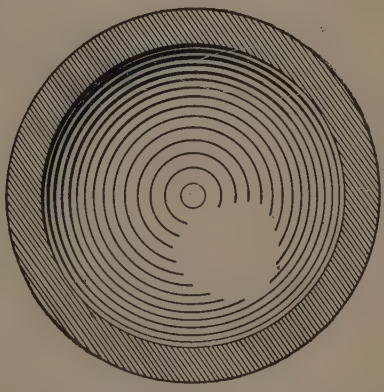

Fig. 179. Shaded Section of Hollow Sphere

center toward each side, thus showing that the cylinder is convex. The effect is intensified, however, if the elements are made heavier, as well as closer together, as shown in Figs. 175 to 181.

Fig. 175 is a cylinder showing the heaviest shade at the right, a method often used. Considerable practice is necessary to obtain good results; but in this, as in other portions of mechanical drawing,

Fig. 180. Shaded Sphere

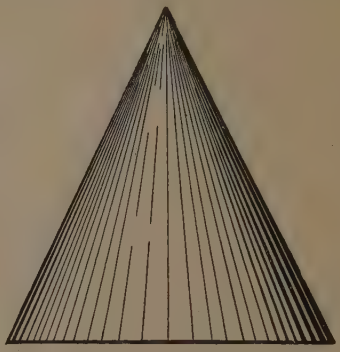

Fig. 181. Shaded Cone

repetition is unavoidable. Fig. 176 represents a cylinder in a horizontal position, and Fig. 177 represents a section of a hollow vertical cylinder. Figs. 178 to 181 give other examples of familiar objects.

In the elevation of the cone shown in Fig. 181 the shade lines should diminish in weight as they approach the apex. Unless this is done it will be difficult to avoid the formation of a blot at that point.

LETTERING

In the early part of this course, the inclined Gothic letter was described, and the alphabet given. The Roman, Gothic, and block letters are perhaps the most used for titles. These letters, being of comparatively large size, are generally made mechanically; that is, drawing instruments are used in their construction. In order that the letters may appear of the same height, some of them, owing to their shape, must be made a little higher than the others. This is the case with the letters curved at the top and bottom, such as C, O, S, etc., as shown somewhat exaggerated in Fig. 182. Also, the letter A should extend a little above, and V a little below, the guide lines, because if made of the same height as the others they will appear shorter. This is true of all capitals, whether of Roman, Gothic, or other alphabets. In the block letter, however, they are frequently all of the same size.

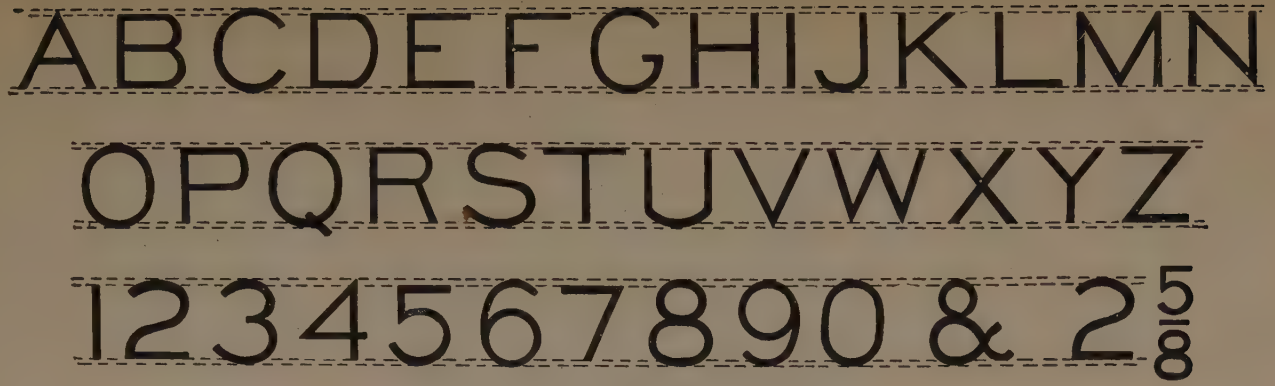

Fig. 182. Vertical Gothic Letters and Figures

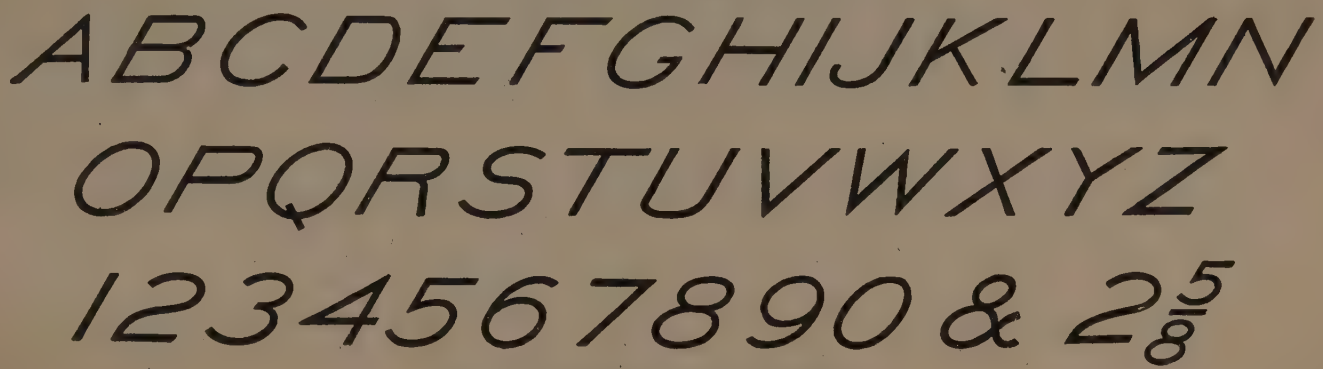

Fig. 183. Inclined Gothic Letters and Figures

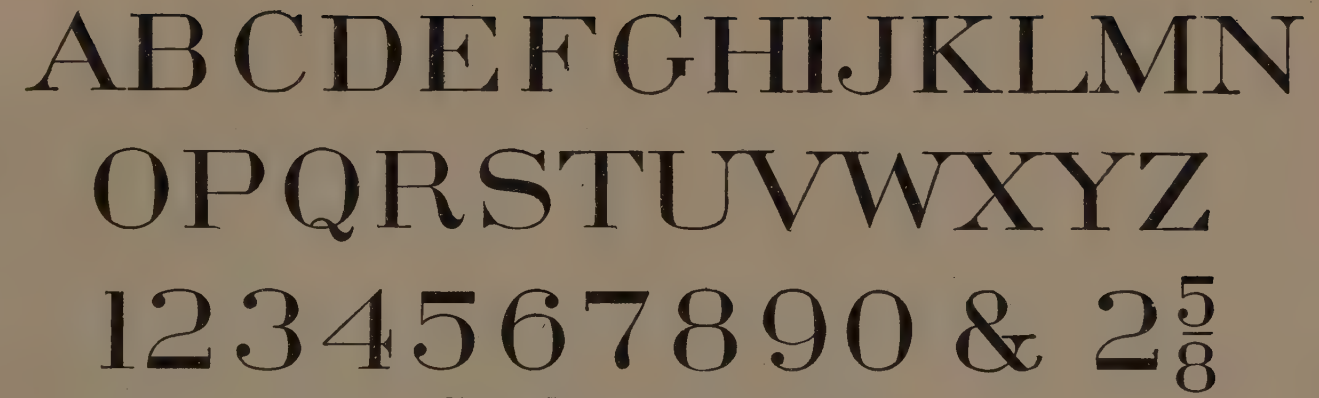

Fig. 184. Vertical Roman Letters and Figures

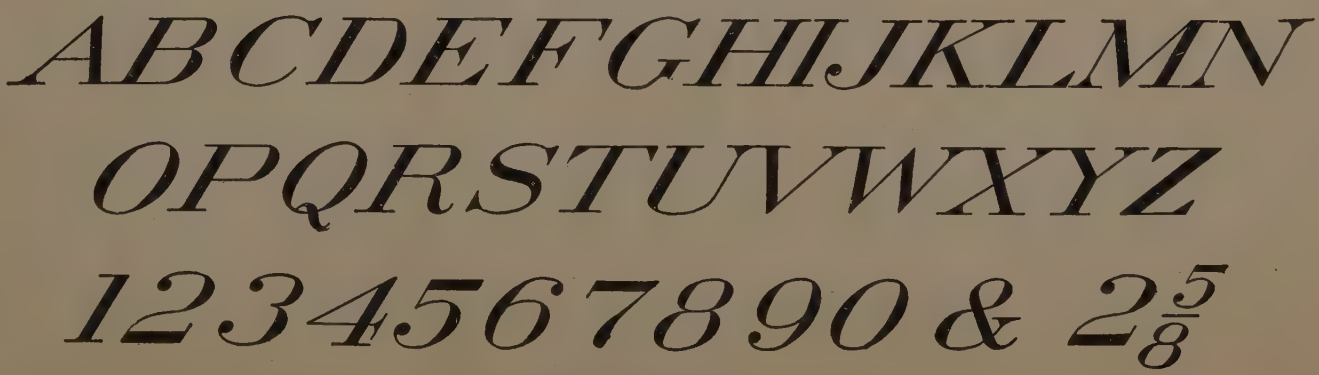

Fig. 185. Inclined Roman Letters and Figures

There is no absolute size or proportion of letters, as the dimensions are regulated by the amount of space in which the letters are to be placed, the size of the drawing, the effect desired, etc. In some cases letters are made so that the height is greater than the width, and sometimes the reverse; sometimes the height and width are the same. This last proportion is the most common. Certain relations of width, however, should be observed. Thus, in whatever style of alphabet used, the W should be the widest letter; J the narrowest, M and T the next widest to W, then A and B. The other letters are of about the same width.

In the vertical Gothic alphabet, the average height is that of B, D, E, F, etc., and the additional height of the curved letters

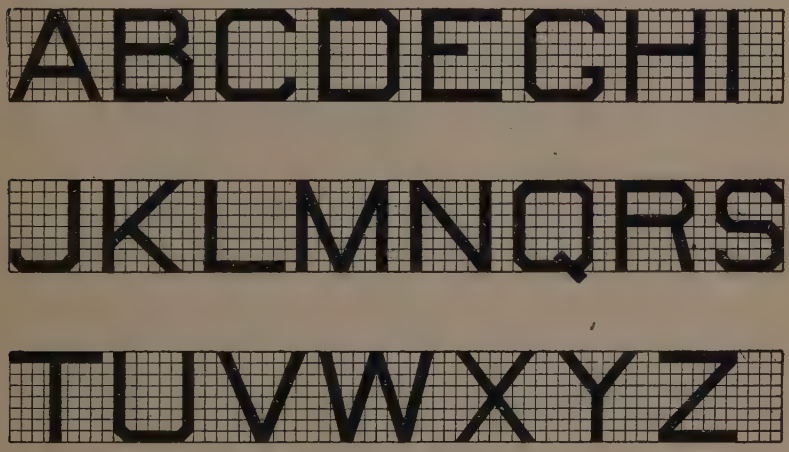

Fig. 186. Block Letters

and of the A and V is very slight. The horizontal cross lines of such letters as E, F, H, etc., are slightly above the center; those of A, G, and P slightly below.

For the inclined letters, Fig. 183, 60° is a convenient angle, although they may be at any other angle suited to the convenience or fancy of the draftsman. Many draftsmen use an angle of about 70°.

The letters of the Roman alphabet, whether vertical, Fig. 184, or inclined, Fig. 185, are quite ornamental in effect if well made, the inclined Roman being a particularly attractive letter, although rather difficult to make. The block letter, Fig. 186, is made on the same general plan as the Gothic, but much heavier. Small squares

are taken as the unit of measurement, as shown. The use of this letter is not advocated for general work, although if made merely in outline the effect is pleasing. The styles of numbers correspond-

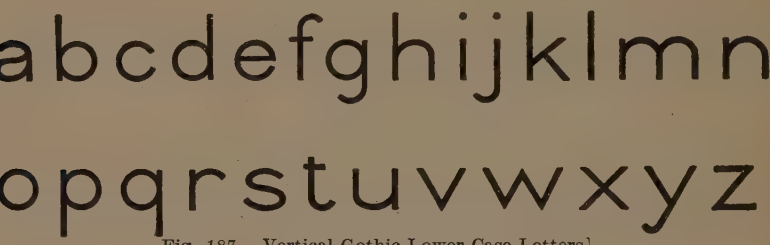

Fig. 187. Vertical Gothic Lower-Case Letters

ing with the alphabets of capitals given here, are also inserted. When a fraction, such as $2\frac{5}{8}$ is to be made, the proportion should be about as shown. For small letters, usually called *lower-case* letters, the

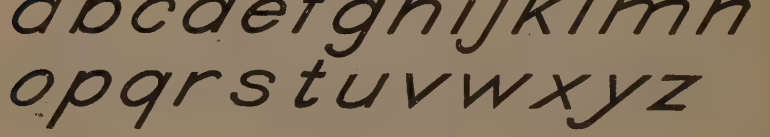

Fig. 188. Inclined Gothic Lower-case Letters

height may be made about two-thirds that of the capitals. This proportion, however, varies in special cases.

The principal lower-case letters in general use among draftsmen are shown in Figs. 187, 188, 189, and 190. The Gothic letters

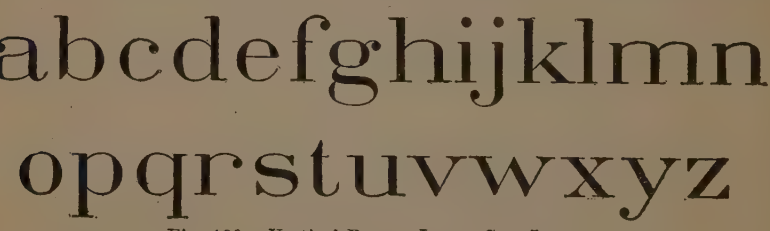

Fig. 189. Vertical Roman Lower-Case Letters

shown in Figs. 187 and 188 are much easier to make than the Roman letters in Figs. 189 and 190. These letters, however, do not give as finished an appearance as the Roman. As has already been stated in Mechanical Drawing, Part I, the inclined letter is easier to make because slight errors are not so apparent.

One of the most important points to be remembered in lettering is the spacing. If the letters are finely executed but poorly spaced, the effect is not good. To space letters correctly and rapidly,

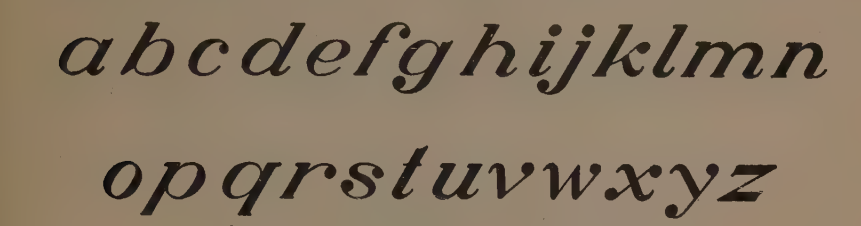

Fig. 190. Inclined Roman Lower-Case Letters

requires considerable experience; and rules are of little value on account of the many combinations in which letters are found. A few directions, however, may be found helpful. For instance, take the word TECHNICALITY, Fig. 191. If all the spaces were made equal, the space between the L and the I would appear to be too great, and the same would apply to the space between the I and the T. The space between the H and the N and that between the N and the I would be insufficient. Usually, when the vertical side of one letter is followed by the vertical side of another, as in H E, H B, I R, etc., the maximum space should be allowed. Where T and A come together the least space is given, for in this case the

Fig. 191. Sample of Letter Spacing

top of the T frequently extends over the bottom of the A. In general, the spacing should be such that a uniform appearance is obtained. For the distances between words in a sentence, a space of about $1\frac{1}{2}$ the width of the average letter may be used. The space, however, depends largely upon the desired effect.

For large titles, such as those placed on charts, maps, and some large working drawings, the letters should be penciled before inking. If the height is made equal to the width considerable time and labor will be saved in laying out the work. This is especially true with such Gothic letters as O, Q, C, etc., as these letters may then be made with compasses. If the letters are of sufficient size,

the outlines may be drawn with the ruling pen or compasses, and the spaces between filled in with a fine brush.

The titles for working drawings are generally placed in the lower right-hand corner. Usually a draftsman has his choice of letters, mainly because after he has become used to making one style he can do it rapidly and accurately. However, in some drafting rooms the head draftsman decides what lettering should be used.

Fig. 192. Sample Title

In making these titles, the different alphabets are selected to give the best results without spending too much time. In most work the letters are made in straight lines, although frequently a portion of the title is found lettered on an arc of a circle.

In Fig. 192 is shown a title having the words CONNECTING ROD lettered on an arc of a circle. To do this work requires considerable patience and practice. First, draw the vertical center line as shown at *C* in Fig. 192; then, draw horizontal lines for the

Fig. 193. Sample Title

horizontal letters. The radii of the arcs depend upon the general arrangement of the entire title, and this is a matter of taste. The difference between the arcs should equal the height of the letters. After the arc is drawn, the letters should be sketched in pencil to find their approximate positions. After this is done, draw radial lines from the center of the letters to the center of the arcs. These lines will be the centers of the letters, as shown at *A*, *B*, *D*, and *E*.

The vertical lines of the letters should not radiate from the center of the arc, but should be parallel to the center lines already drawn; otherwise the letters will appear distorted. Thus, in the letter *N* the two verticals are parallel to the line *A*. The same applies to the other letters in the alphabet. In making the curved letters such as O and C, the centers of the arcs will fall upon these center lines; and if the compasses are used, the lettering is a comparatively simple matter. In Fig. 193 is shown another title in which all the letters are in horizontal lines.

PROBLEMS IN PROJECTION

PLATE IX

The plates of this Instruction Paper should be laid out the same size as the plates in Parts I and II. The center lines and border lines should also be drawn as described.

Draw two ground lines across the sheet, one 3 inches below the upper border line and the other 3 inches above the lower border line. The first problem on each ground line is to be placed 1 inch from the left border line; and spaces of about 1 inch should be left between the figures.

Isolated points are indicated by a small cross, x, and projections of lines are to be drawn full unless invisible. All construction lines should be fine dotted lines. Given and required lines should be drawn full.

Problems on Upper Ground Line:

1. Locate both projections of a point on the horizontal plane 1 inch from the vertical plane.

2. Draw the projections of a line 2 inches long which is parallel to the vertical plane and which makes an angle of 45 degrees with the horizontal plane and slants upward to the right.

Note. The line should be 1 inch from the vertical plane and the lower end $\frac{1}{2}$ inch above the horizontal.

3. Draw the projections of a line $1\frac{1}{2}$ inches long which is parallel to both planes, 1 inch above the horizontal, and $\frac{3}{4}$ inch from the vertical.

4. Draw the plan and elevation of a line 2 inches long which is parallel to H and makes an angle of 30 degrees with V. Let the right-hand end of the line be the end nearer V, $\frac{1}{2}$ inch from V. The line to be 1 inch above H.

5. Draw the plan and elevation of a line $1\frac{1}{2}$ inches long which is perpendicular to the horizontal plane and 1 inch from the vertical. Lower end of line is $\frac{1}{2}$ inch above H.

6. Draw the projections of a line 1 inch long which is perpendicular to the vertical plane and $1\frac{1}{2}$ inches above the hori-

zontal. The end of the line nearer V, or the back end, is $\frac{1}{2}$ inch from V.

7. Draw two projections which shall represent a line oblique to both planes.

NOTE. Leave 1 inch between this figure and the right-hand border line.

Problems on Lower Ground Lines:

8. Draw the projections of two parallel lines each $1\frac{1}{2}$ inches long. The lines are to be parallel to the vertical plane and to make angles of 60 degrees with the horizontal. The lower end of each line is $\frac{1}{4}$ inch above H. The right-hand end of the right-hand line is to be $2\frac{3}{4}$ inches from the left-hand margin.

9. Draw the projections of two parallel lines each 2 inches long. Both lines to be parallel to the horizontal and to make an angle of 30 degrees with the vertical. The lower line to be $\frac{3}{4}$ inch above H, and one end of one line to be against V.

10. Draw the projections of two intersecting lines. One 2 inches long to be parallel to both planes, 1 inch above H, and $\frac{3}{4}$ inch from the vertical; and the other to be oblique to both planes and of any desired length.

11. Draw plan and elevation of a prism 1 inch square and $1\frac{1}{2}$ inches long. The prism to have one side on the horizontal plane, and its long edges to be perpendicular to V. The back end of the prism is $\frac{1}{4}$ inch from the vertical plane.

12. Draw plan and elevation of a prism the same size as given above, but with the long edges parallel to both planes, the lower face of the prism to be parallel to H and $\frac{1}{4}$ inch above it. The back face to be $\frac{1}{2}$ inch from V.

PLATE X

The ground line is to be in the middle of the sheet, and the location and dimensions of the figures are to be as given. The first figure shows a rectangular block with a rectangular hole cut through from front to back. The other two figures represent the same block in different positions. The second figure is the end or profile projection of the block. The same face is on H in all three positions. The figures given on the plate for dimensions, etc., are to be used but not repeated on the plate by the student.

PLATE XI

Three ground lines are to be used on this plate, two at the left $4\frac{1}{2}$ inches long and 3 inches from top and bottom margin lines; and one at the right, half way between the top and bottom margins, $9\frac{1}{2}$ inches long.

The figures 1, 2, 3, and 4 are examples for finding the true lengths of the lines. Begin No. 1 at $\frac{3}{4}$ inch from the border, the vertical projection $1\frac{3}{4}$ inches long, one end on the ground line and inclined at 30°. The horizontal projection has one end $\frac{1}{2}$ inch from *V*, and the other $1\frac{1}{2}$ inches from *V*. Find the true length of the line by completing the construction commenced by swinging the arc, as shown in the figure.

Locate the left-hand end of No. 2 at 3 inches from the border, 1 inch above *H*, and $\frac{5}{8}$ inch from *V*. Extend the vertical projection to the ground line at an angle of 45°, and make the horizontal projection at 30°. Complete the construction for true length as commenced in the figure.

In Figs. 3 and 4, the true lengths are to be found by completing the revolutions indicated. The left-hand end of Fig. 3 is $\frac{3}{4}$ inch from the margin, $1\frac{1}{2}$ inches from *V*, and $1\frac{3}{8}$ inches above *H*. The horizontal projection makes an angle of 60° and extends to the ground line, and the vertical projection is inclined at 45°.

The fourth figure is 3 inches from the border, and represents a line in a profile plane connecting points *a* and *b*. *a* is $1\frac{1}{4}$ inches above *H* and $\frac{3}{4}$ inch from *V;* and *b* is $\frac{1}{4}$ inch above *H* and $1\frac{1}{2}$ inches from *V*.

The figures for the middle ground line represent a pentagonal pyramid in three positions. The first position is the pyramid with the axis vertical, and the base $\frac{5}{8}$ inch above the horizontal. The height of the pyramid is $2\frac{1}{2}$ inches, and the diameter of the circle circumscribed about the base is $2\frac{1}{2}$ inches. The center of the circle is 6 inches from the left margin and $1\frac{3}{4}$ inches from *V*. Spaces between figures to be $\frac{3}{4}$ inch.

In the second figure the pyramid has been revolved about the right-hand corner of the base as an axis, through an angle of 15°. The axis of the pyramid, shown dotted, is therefore at 75°. The method of obtaining 75° and 15° with the triangles was shown in

Part I. From the way in which the pyramid has been revolved, all angles with V must remain the same as in the first position; hence the vertical projection will be the same shape and size as before. All points of the pyramid remain the same distance from V. The points on the plan are found on T-square lines through the corners of the first plan and directly beneath the points in elevation. In the third position the pyramid has been swung around, about a vertical line through the apex as axis, through 30°. The angle with the horizontal plane remains the same; consequently the plan is the same size and shape as in the second position, but at a different angle with the ground line. Heights of all points of the pyramid have not changed this time, and hence are projected across from the second elevation.

PLATE XII

Developments

On this plate draw the developments of a truncated octagonal prism, and of a truncated pyramid having a square base. The

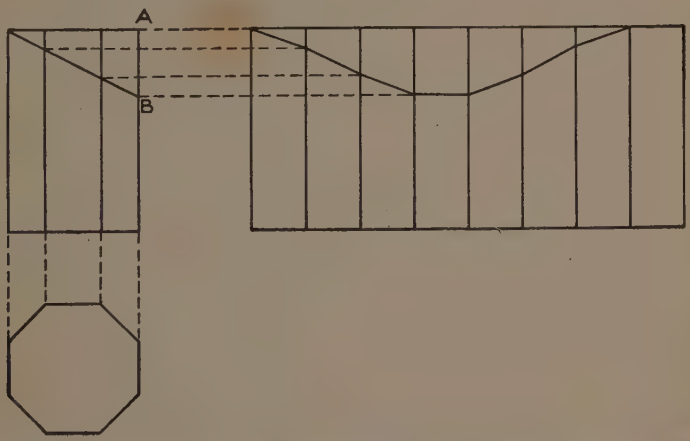

Fig. 194. Plan, Elevation, and Development of an Octagonal Truncated Prism

arrangement on the plate is left to the student; but it is suggested that the truncated prism and its development be placed at the left, and that the development of the truncated pyramid be placed under the development of the prism; the truncated pyramid may be placed at the right.

The prism and its development are shown in Fig. 194. The

prism is 3 inches high, and the base is inscribed in a circle $2\frac{1}{8}$ inches in diameter. The plane forming the truncated prism is passed as indicated, the distance $A\ B$ being 1 inch. Ink a sufficient number

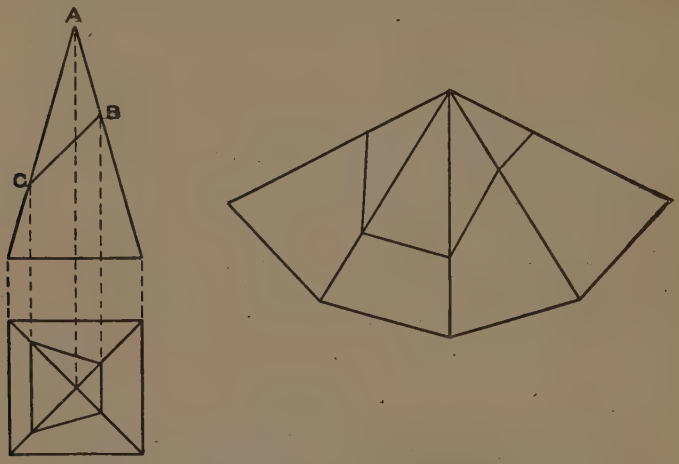

Fig. 195. Plan, Elevation, and Development of a Square Pyramid and Cutting Plane

of construction lines to show clearly the method of finding the development.

The pyramid and its development are shown in Fig. 195. Each side of the square base is 2 inches, and the altitude is $3\frac{1}{2}$ inches. The plane forming the truncated pyramid is passed in such a position that $A\ B$ equals $1\frac{3}{8}$ inches, and $A\ C$ equals $2\frac{1}{2}$ inches. In this figure the development may be drawn in any convenient position, but in the case of the prism it is better to draw the development as shown. Indicate clearly the construction by inking the construction lines.

PLATE XIII

Isometric and Oblique Projection

Draw the oblique projection of a portable closet. The angle to be used is 45°. Make the height $3\frac{1}{2}$ inches, the depth $1\frac{1}{2}$ inches, and the width 3 inches. See Fig. 196. The width of the closet is to be shown as the right-hand face. The front left-hand lower corner is to be 1 inch from the left-hand border line and 2 inches from the lower border line. The door to be placed in the closet should be $1\frac{5}{8}$ inches wide and $2\frac{3}{4}$ inches high. Place the door centrally in the front of the closet, the bottom edge at the height of

the floor of the closet, the hinges of the door to be placed on the left-hand side. In the oblique drawing, show the door opened at an angle of 90 degrees. The thickness of the material of the closet,

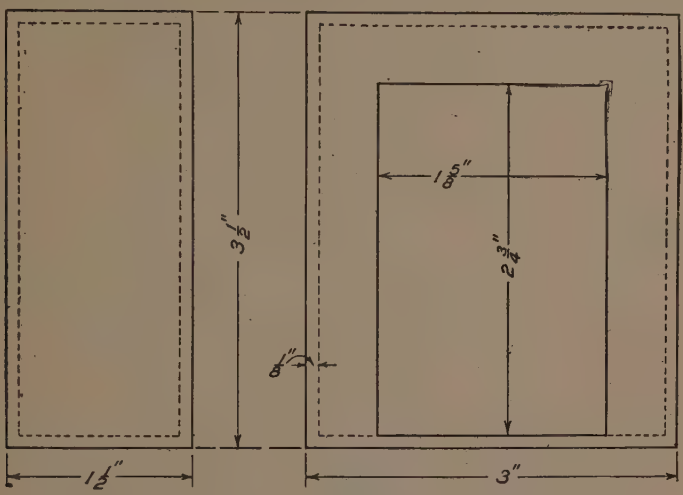

Fig. 196. End View and Front Elevation of a Portable Closet

door, and floor is $\frac{1}{8}$ inch. The door should be hung so that when closed it will be flush with the front of the closet.

Make the isometric drawing of the flight of steps and end walls as shown by the end view in Fig. 197. The lower right-hand corner is to be located $2\frac{1}{4}$ inches from the lower, and 5 inches from the right-hand, margin. The base of the end wall is $3\frac{1}{8}$ inches long, and the height is $2\frac{1}{4}$ inches. Beginning from the back of the wall, the top is horizontal for $\frac{5}{8}$ inch, the remainder of the outline being composed of arcs of circles whose radii and centers are given in the figure. The thickness of the end wall is $\frac{3}{8}$ inch, and both ends are alike. There are to be five steps; each rise is to be $\frac{3}{8}$ inch, and each tread $\frac{1}{2}$ inch, except that of the top step, which is $\frac{3}{4}$ inch. The first step is located $\frac{3}{8}$ inch back from the corner of the wall. The end view of the wall

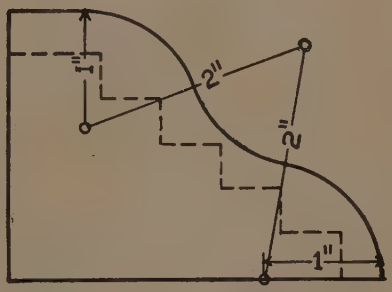

Fig 197. End View of Flight of Steps

should be constructed on a separate sheet of paper, from the dimensions given, the points on the curve being located by horizontal co-ordinates from the vertical edge of the wall, and then these co-ordinates transferred to the isometric drawing. After the isometric of one curved edge has been made, the others can be readily found from this. The width of the steps inside the walls is 3 inches.

PLATE XIV

Free-hand Lettering

On account of the importance of free-hand lettering, the student should practice it at every opportunity. For additional practice, and to show the improvement made since completing Part I, lay out Plate XIV in the same manner as Plate I, and letter all four rectangles. Use the same letters and words as in the lower right-hand rectangle of Plate I.

PLATE XV

Lettering

First lay out Plate XV in the same manner as previous plates. After drawing the vertical center line, draw light pencil lines as guide lines for the letters. The height of each line of letters is shown on the reproduced plate. The distance between the letters should be $\frac{1}{2}$ inch in every case. The spacing of the letters is left to the student. He may facilitate his work by lettering the words on a separate piece of paper, and finding the center by measurement or by doubling the paper into two equal parts. The styles of letters shown on the reproduced plate should be used.

PLATE X

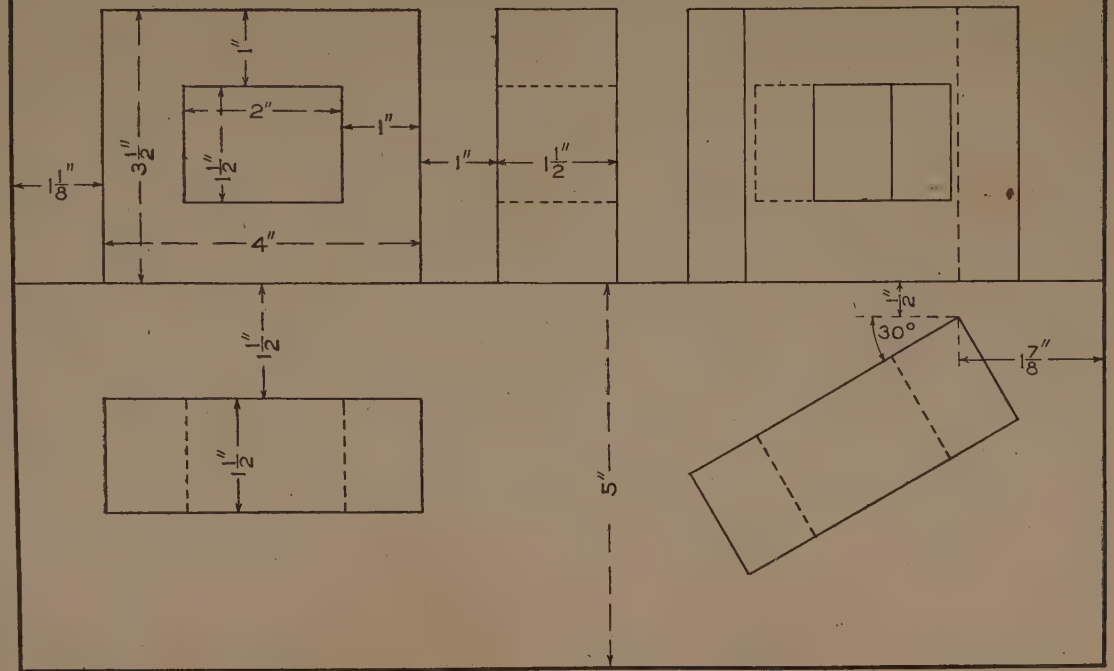

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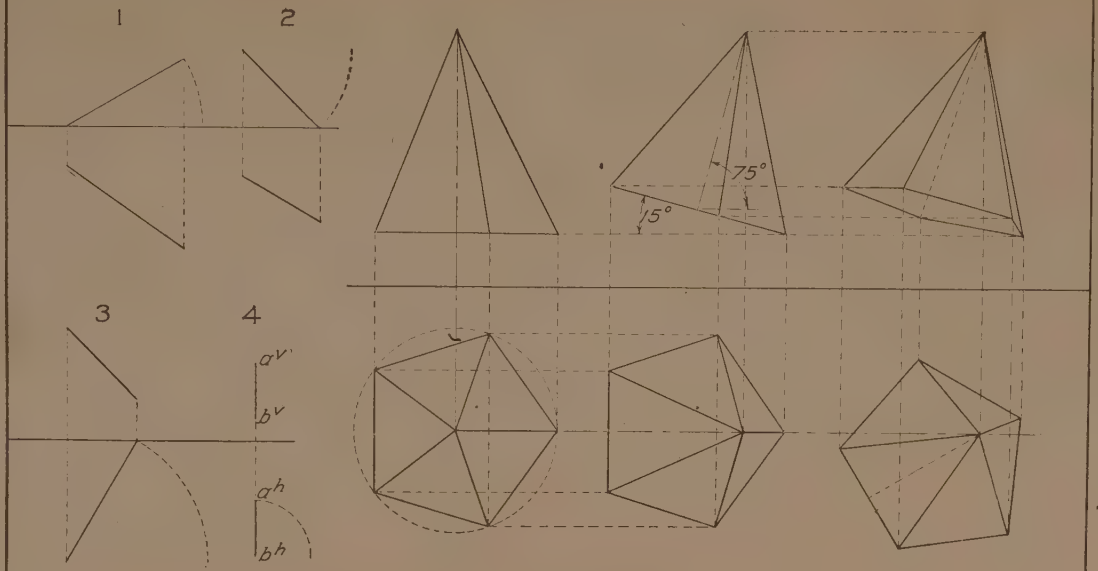
PLATE XI



FEBRUARY 27, 1910. HERBERT CHANDLER, CHICAGO, ILL.

COURSE
IN
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AMERICAN SCHOOL
OF
CORRESPONDENCE
CHICAGO, ILL., U.S.A.

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